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Automatic Portmanteau Tests with Applications to Market Risk Management

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Abstract

This article reviews some recent advances in testing for serial correlation, provides Stata code for implementation and illustrates its application to market risk forecast evaluation. The classical and widely used Portmanteau tests and their data-driven versions are the focus of this article. These tests are simple to implement for two reasons: first, the researcher does not need to specify the order of the autocorrelation tested, since the test automatically chooses this number; second, its asymptotic null distribution is chi-square with one degree of freedom, so there is no need of using a bootstrap procedure to estimate the critical values. We illustrate the wide applicability of the methodology with applications to forecast evaluation for market risk measures, such as Value-at-Risk and Expected Shortfall.

Keywords and Phrases: Autocorrelation, consistency, power, Akaike's AIC, Schwarz's BIC, Market Risk

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1. INTRODUCTION

Testing for serial correlation has held a central role in time series analysis since its inception (see the early contributions by Yule (1926) and Quenouille (1947)). Despite the many proposals and variations since the seminal contribution of Box and Pierce (1970), still the so-called Portmanteau tests are the most widely used. In its simplest form, the employed statistic is just the sample size times the sum of the first p squared sample autocorrelations, which is compared with critical values from a chi-square distribution with p degrees of freedom (with a correction if the test is applied to residuals). The basic Box-Pierce statistic has been slightly modified to improve its finite sample performance, see Davies, Triggs and Newbold (1977), Ljung and Box (1978), Davies and Newbold (1979) or Li and McLeod (1981). The properties of the classical Box-Pierce tests have been extensively studied in the literature; see e.g. the monograph by Li (2004) for a review of this literature. Much of the theoretical literature on Box-Pierce tests was developed under the independence assumption, and hence is generally invalid when applied to dependent data (the asymptotic size of the test is different from the nominal level); see Newbold (1980) or more recently Francq, Roy and Zakoian (2005) for valid tests. This limitation of classical Box-Pierce tests is by now well understood. This paper focuses on another limitation of Classical Box-Pierce tests: the selection of the employed number of autocorrelations is arbitrary. We review the contribution of Escanciano and Lobato (2009), who proposed a data-driven Portmanteau statistic where the number of correlations is not fixed but selected automatically from the data. In this paper, we give a synthesis of this methodology, introduce new general assumptions for its validity, review new applications in risk management and provide Stata code for its implementation.

2. AUTOMATIC PORTMANTEAU TESTS: A SYNTHESIS

Given a strictly stationary process $\{Y_t\}_{t \in \mathbb{Z}}$ with $E[Y_t^2] < \infty$ and $\mu = E[Y_t]$, define the autocovariance of order j as

$$\gamma_j = \text{Cov}(Y_t, Y_{t-j}) = E[(Y_t - \mu)(Y_{t-j} - \mu)], \quad \text{for all } j \geq 0,$$

and the j -th order autocorrelation as $\rho_j = \gamma_j / \gamma_0$. We aim to test the null hypothesis

$$H_0 : \rho_j = 0, \quad \text{for all } j \geq 1,$$

against the fixed alternative hypotheses

$$H_1^K : \rho_j \neq 0, \quad \text{for some } 1 \leq j \leq K, \tag{1}$$

and some $K \geq 1$.

Suppose we observe data $\{Y_t\}_{t=1}^n$. Then, γ_j can be consistently estimated by the sample autocovariance

$$\hat{\gamma}_j = \frac{1}{(n-j)} \sum_{t=1+j}^n (Y_t - \bar{Y})(Y_{t-j} - \bar{Y}), \quad j = 0, \dots, n-1,$$

where $\bar{Y}$ is the sample mean, and also introduce $\hat{\rho}_j = \hat{\gamma}_j / \hat{\gamma}_0$ to denote the j -th order sample autocorrelation.

The Box-Pierce Q_p statistic (cf. Box and Pierce, 1970) is just

$$Q_p = n \sum_{j=1}^p \hat{\rho}_j^2,$$

which is commonly implemented via the Ljung and Box (1978) modification

$$LB_p = n(n+2) \sum_{j=1}^p (n-j)^{-1} \hat{\rho}_j^2. \quad (2)$$

When $\{Y_t\}_{t=1}^n$ is independent and identically distributed (iid), both Q_p and LB_p converge to a chi-squared distribution with p degrees of freedom, in short χ_p^2 . When $\{Y_t\}_{t=1}^n$ is serially dependent, like for example when Y_t is a residual from a fitted model, the asymptotic distribution Q_p and LB_p is generally different from χ_p^2 and depends on the data generating process in a complicated way, see for instance Francq, Roy and Zakoïan (2005) and Delgado and Velasco (2011).

In this section we provide a synthesis of the automatic Portmanteau test methodology suggested in Escanciano and Lobato (2009), thereby extending the methodology to other situations. The main ingredients of the methodology are: (1) the following asymptotic results for individual autocorrelations, for $j = 1, \dots, d$, with d is a fixed upper bound,

$$\sqrt{n} (\hat{\rho}_j - \rho_j) \xrightarrow{d} N(0, \tau_j), \quad (3)$$

for a positive asymptotic variance $\tau_j > 0$, with

$$\hat{\tau}_j \xrightarrow{p} \tau_j; \quad (4)$$

and (2) a data-driven construction of p given below. For *iid* observations $\tau_j = 1$, and trivially we can take $\hat{\tau}_j = 1$, but in other more general settings with weak dependence or estimation effects we will have an unknown $\tau_j \neq 1$ that needs to be estimated. Our definitions of Portmanteau tests allow for general cases. Define

$$Q_p^* = n \sum_{j=1}^p \tilde{\rho}_j^2,$$

where $\tilde{\rho}_j = \hat{\rho}_j / \sqrt{\hat{\tau}_j}$ is called a *Generalized Autocorrelation* here. Then, the Automatic Portmanteau test is given by

$$AQ = Q_p^* \quad (5)$$

where

$$\tilde{p} = \min\{p : 1 \leq p \leq d; L_p \geq L_h, h = 1, 2, \dots, d\},$$

with

$$L_p = Q_p^* - \pi(p, n, q),$$

$\pi(p, n, q)$ is a penalty term that takes the form

$$\pi(p, n, q) = \begin{cases} p \log n, & \text{if } \max_{1 \leq j \leq d} \sqrt{n} |\tilde{\rho}_j| \leq \sqrt{q \log n}, \\ 2p, & \text{if } \max_{1 \leq j \leq d} \sqrt{n} |\tilde{\rho}_j| > \sqrt{q \log n}, \end{cases} \quad (6)$$

and $q = 2.4$. Theoretical and empirical motivation for the choice of $q = 2.4$ is provided in Inglot and Ledwina (2006b) and Escanciano and Lobato (2009). This choice provides a “switching effect” in which one combines the advantages of the two selection rules involved, Akaike Information Criterion (AIC), see Akaike (1974), and the Bayesian Information Criterion (BIC), see Schwarz (1978). This penalty term has been previously considered by Inglot and Ledwina (2006a) for Neyman’s original problem of testing for goodness of fit for a distribution. Kallenberg (2002) discusses alternative choices for $\pi(p, n, q)$.

THEOREM 1: *Under the null hypothesis, (3) and (4), $AQ \xrightarrow{d} \chi_1^2$.*

This Theorem justifies the rejection region

$$AQ > \chi_{1,1-\alpha}^2,$$

where $\chi_{1,1-\alpha}^2$ is the $(1 - \alpha)$ -quantile of the χ_1^2 . The following Theorem shows the consistency of the test.

THEOREM 2: *Assume $\hat{\rho}_j \xrightarrow{p} \rho_j$ for $j = 1, \dots, d$, and let (4) hold. Then, the test based on AQ is consistent against H_1^K , for $K \leq d$.*

The proofs of both theorems follow from straightforward modification of those in Escanciano and Lobato (2009), and hence they are omitted.

REMARK 1. The methodology can be applied to any setting where (3) and (4) can be established. This includes raw data or residuals from any model. There is an extensive literature proving conditions such as (3) and (4) under different assumptions, see examples below. Note that joint convergence of the vector of autocorrelations is not needed, in contrast to much of the literature. Thus, the methodology of this paper does not require estimation of large dimensional asymptotic variances.

REMARK 2. The reason for the χ_1^2 limiting distribution of the Automatic Portmanteau test is that under the null hypothesis $\lim_{n \rightarrow \infty} P(\tilde{p} = 1) = 1$. Heuristically, under the null Q_p^* is small, and $\pi(p, n, q)$ increases in p , so the optimal choice selected is the lowest dimensionality $p = 1$ with high probability.

3. APPLICATIONS TO RISK MANAGEMENT

We illustrate the general applicability of the methodology with new applications in risk management. There is a very extensive literature on the quantification of market risk for derivative pricing, for portfolio choice and for risk management purposes. In particular, this literature has long been interested in the evaluation of market risk forecasts, the so-called backtests, see Jorion (2006) and Christoffersen (2009) for comprehensive reviews. A leading market risk measure has been the Value at Risk (VaR), and more recently Expected Shortfall (ES). VaR summarizes the worst loss over a target horizon that will not be exceeded at a given level of confidence called coverage level. ES is the expected value of losses beyond a given level of confidence.¹ We review popular backtests for VaR and ES and derive automatic versions using the general methodology above.

Let R_t denote the revenue of a bank at time t , and let Ω_{t-1} denote the risk manager's information at time $t-1$, which contains lagged values of R_t and possibly lagged values of other variables, say X_t . That is, $\Omega_{t-1} = \{X_{t-1}, X_{t-2}, \dots; R_{t-1}, R_{t-2}, \dots\}$. Let $G(\cdot, \Omega_{t-1})$ denote the conditional cumulative distribution function (*cdf*) of R_t given Ω_{t-1} , i.e. $G(\cdot, \Omega_{t-1}) = \Pr(R_t \leq \cdot | \Omega_{t-1})$. Assume $G(\cdot, \Omega_{t-1})$ is continuous. Let $\alpha \in [0, 1]$ denote the coverage level. The α -level VaR is defined as the quantity $VaR_t(\alpha)$ such that

$$\Pr(R_t \leq -VaR_t(\alpha) | \Omega_{t-1}) = \alpha. \quad (7)$$

That is, the $-VaR_t(\alpha)$ is the α -th percentile of the conditional distribution G ,

$$VaR_t(\alpha) = -G^{-1}(\alpha, \Omega_{t-1}) = -\inf\{y : G(y, \Omega_{t-1}) \geq \alpha\}.$$

Define the α -violation or hit at time t as

$$h_t(\alpha) = 1(R_t \leq -VaR_t(\alpha)),$$

where $1(\cdot)$ denotes the indicator function. That is, the violation takes the value one if the loss at time t is larger than or equal to $VaR_t(\alpha)$, and it is zero otherwise. An implication of (7) is that violations are Bernoulli variables with mean α , and moreover, centered violations are a martingale difference sequence (*mds*) for each $\alpha \in [0, 1]$, i.e.

$$E[h_t(\alpha) - \alpha | \Omega_{t-1}] = 0 \text{ for each } \alpha \in [0, 1].$$

¹Other names for ES are Conditional VaR, Average VaR, tail VaR or expected tail loss.

This restriction has been the basis for the extensive literature on backtesting VaR . Two of its main implications, the zero mean property of the hit sequence $\{h_t(\alpha) - \alpha\}_{t=1}^\infty$ and its uncorrelation led to the unconditional and conditional backtests of Kupiec (1995) and Christoffersen (1998), respectively, which are the most widely used backtests. More recently, Berkowitz, Christoffersen and Pelletier (2011) have proposed the Box-Pierce-type test for VaR

$$C_{VaR}(p) = n \sum_{j=1}^p \hat{\rho}_j^2,$$

with $\hat{\rho}_j = \hat{\gamma}_j / \hat{\gamma}_0$ and $\hat{\gamma}_j = 1/(n-j) \sum_{t=1+j}^n (\hat{h}_t(\alpha) - \alpha)(\hat{h}_{t-j}(\alpha) - \alpha)$, and where $\{\hat{h}_t(\alpha) = R_t \leq -\widehat{VaR}_t(\alpha)\}_{t=1}^n$, for an estimator of the VaR , $\widehat{VaR}_t(\alpha)$. An automatic version of the test statistic in Berkowitz, Christoffersen and Pelletier (2011) can be computed following the algorithm above with $\tau_j = 1$. This test is only valid when there is no estimation effects (e.g. when the in-sample size is much larger than the out-of-sample size). More generally, Escanciano and Olmo (2010) provided primitive conditions for the convergences (3) and (4) to hold in a general setting where there is estimating effects from estimating VaR . When estimation effects are present τ_j no longer equals 1, but Escanciano and Olmo (2010) provide suitable estimators $\hat{\tau}_j$ satisfying (4). Let AC_{VaR} denote the Automatic Portmanteau version of $C_{VaR}(p)$.

More recently there has been a move in the banking sector towards ES as a suitable measure of market risk that is able to capture “tail risk” (the risk coming from very big losses). ES is defined as the conditional expected loss given that the loss is larger than $VaR_t(\alpha)$, that is,

$$ES_t(\alpha) = E[-R_t | \Omega_{t-1}, -R_t > VaR_t(\alpha)]. \quad (8)$$

Definition of a conditional probability and a change of variables yield a useful representation of $ES_t(\alpha)$ in terms of $VaR_t(\alpha)$,

$$ES_t(\alpha) = \frac{1}{\alpha} \int_0^\alpha VaR_t(u) du. \quad (9)$$

Unlike $VaR_t(\alpha)$, which only contains information on one quantile level α , $ES_t(\alpha)$ contains information from the whole left tail, by integrating all VaR s from 0 to α . In analogy with (9), we define the cumulative violation process,

$$H_t(\alpha) = \frac{1}{\alpha} \int_0^\alpha h_t(u) du.$$

Since $h_t(u)$ has mean u , by Fubini's Theorem $H_t(\alpha)$ has mean $1/\alpha \int_0^\alpha u du = \alpha/2$. Moreover, again by Fubini's Theorem, the *mds* property of the class $\{h_t(\alpha) - \alpha : \alpha \in [0, 1]\}_{t=1}^\infty$ is preserved by integration, which means that $\{H_t(\alpha) - \alpha/2\}_{t=1}^\infty$ is also a *mds*. For computational purposes, it is convenient to define $u_t = G(R_t, \Omega_{t-1})$. Using that $h_t(u) = 1(R_t \leq -VaR_t(u)) = 1(u_t \leq u)$, we

obtain

$$\begin{aligned} H_t(\alpha) &= \frac{1}{\alpha} \int_0^\alpha 1(u_t \leq u) du \\ &= \frac{1}{\alpha} (\alpha - u_t) 1(u_t \leq \alpha). \end{aligned} \quad (10)$$

Like violations, cumulative violations are distribution-free, since $\{u_t\}_{t=1}^\infty$ comprises a sample of *iid* $U[0, 1]$ variables (see Rosenblatt (1952)). Cumulative violations have been recently introduced in Du and Escanciano (2017). The variables $\{u_t\}_{t=1}^\infty$ necessary to construct $\{H_t(\alpha)\}_{t=1}^\infty$ are generally unknown, since the distribution of the data G is unknown. In practice, researchers and risk managers specify a parametric conditional distribution $G(\cdot, \Omega_{t-1}, \theta_0)$, where θ_0 is some unknown parameter in $\Theta \subset \mathbb{R}^p$, and proceed to estimate θ_0 before producing *VaR/ES* forecasts. Popular choices for distributions $G(\cdot, \Omega_{t-1}, \theta_0)$ are those derived from location-scale models with Student's t distributions, but other choices can be certainly entertained in our setting. With the parametric model at hand, we can define the “generalized errors”

$$u_t(\theta_0) = G(R_t, \Omega_{t-1}, \theta_0)$$

and the associated cumulative violations

$$H_t(\alpha, \theta_0) = \frac{1}{\alpha} (\alpha - u_t(\theta_0)) 1(u_t(\theta_0) \leq \alpha).$$

Very much like for *VaRs*, the arguments above provide a theoretical justification for backtesting *ES* by checking whether $\{H_t(\alpha, \theta_0) - \alpha/2\}_{t=1}^\infty$ have zero mean (unconditional *ES* backtest) and whether $\{H_t(\alpha, \theta_0) - \alpha/2\}_{t=1}^\infty$ are uncorrelated (conditional *ES* backtest).

Let $\hat{\theta}$ be an estimator of θ_0 and construct residuals

$$\hat{u}_t = G(R_t, \Omega_{t-1}, \hat{\theta}),$$

and estimated cumulative violations

$$\hat{H}_t(\alpha) = \frac{1}{\alpha} (\alpha - \hat{u}_t) 1(\hat{u}_t \leq \alpha).$$

Then, we obtain

$$\hat{\gamma}_j = \frac{1}{n-j} \sum_{t=1+j}^n (\hat{H}_t(\alpha) - \alpha/2)(\hat{H}_{t-j}(\alpha) - \alpha/2) \text{ and } \hat{\rho}_j = \frac{\hat{\gamma}_j}{\hat{\gamma}_0}.$$

Du and Escanciano (2017) construct the Box-Pierce test statistic

$$C_{ES}(p) = n \sum_{j=1}^p \hat{\rho}_j^2, \quad (11)$$

and derive its asymptotic null distribution. In particular, they establish conditions for (3) and (4) to hold and provide expressions for the corresponding $\hat{\tau}_j$. Let AC_{ES} denote the Automatic Portmanteau version of $C_{ES}(p)$.

Compared to the existing backtests, these automatic backtests select p from the data, and only require estimation of marginal asymptotic variances of marginal correlations to obtain known limiting distributions.

4. STATA IMPLEMENTATION

We introduce the `dbptest` command to implement the automatic portmanteau test (5). Notice that $\tau_j = 1$ for *iid* observations, as well as backtesting *VaR* and *ES* without estimation effects.

We also provide a Stata command `rtau` to estimate τ_j for more general cases, including martingale difference sequence as in Escanciano and Lobato (2009), as well as backtests for *VaR* and *ES* with estimation effects as in Escanciano and Olmo (2010) and Du and Escanciano (2017), respectively.

4.1 Syntax

Automatic Q Test.—

```
dbptest varname [if] [in] [, mu(#) q(#) tauvector(matname) nlags(#)]
```

Estimating τ_j .—

```
rtau varname [if] [in] , nlags(#) seriestype(type) [cl(#) nobs(#)]
```

4.2 Options

Automatic Q Test.—

`mu(#)` specifies the mean of the variable tested. The default is the variable's sample mean.

`q(#)` is some fixed positive number to control the switching effect between AIC and BIC. The default value is 2.4.

`tauvector(matname)` specifies a column vector containing variances of the autocorrelations. The default is a vector of 1's.

`nlags(#)` specifies the maximum number of lags of autocorrelations. The default is the closest integer around $\sqrt{n}$, where n is the number of observations. If it is larger than the dimension of `tauvector`, it will be replaced by the dimension of `tauvector`.

Estimating τ_j .—

`nlags(#)` specifies the number of lags of autocorrelations.

`seriestype(type)` specifies one of the following three types: *mds*, *var* and *es*.

`seriestype(mds)` specifies *varname* to be a martingale difference sequence as in Escanciano and Lobato (2009).

`seriestype(var)` corresponds to backtesting *VaR*. *varname* assumes an AR(1)-GARCH(1,1) model with student-*t* innovations, when deriving the estimation effects.

`seriestype(es)` corresponds to backtesting *ES*. *varname* assumes an AR(1)-GARCH(1,1) model with student-*t* innovations, when deriving the estimation effects.

`c1(#)` specifies the coverage level of *VaR* and *ES*. The default is 0.05.

`nobs(#)` specifies the in-sample size when backtesting *VaR* and *ES*.

4.3 Remarks

One needs to `tsset` the data before using `dbptest` and `rtau`.

Automatic Q Test.—

`dbptest` implements a data-driven Box-Pierce test for serial correlations. The test automatically chooses the order of autocorrelations. The command reports not only the usual outputs of Box-Pierce test as `wntestq`, i.e., the Q statistics and the corresponding P-value, but also the automatic number of lags chosen.

Estimating τ_j .—

`rtau` estimates the asymptotic variances of autocorrelations when necessary. This includes

- (1) martingale difference sequence data;
- (2) backtesting *ES* and *VaR* with estimation effects.

Only when *var* or *es* is specified, `c1(#)` and `nobs(#)` are required.

4.4 Saved results

Automatic Q Test.—

Scalars

<code>r(stat)</code>	Q statistic	<code>r(p)</code>	probability value
<code>r(lag)</code>	the number of lag(s)		

Estimating τ_j .—

Matrix

<code>e(tau)</code>	variances of autocorrelations
---------------------	-------------------------------

4.5 Example

To illustrate the usage of the two commands, we consider the DAX Index return data from January 1, 1997 to June 30, 2009 as in Du and Escanciano (2017). The in-sample period is from January 1, 1997, to June 30, 2007. The out-of-sample period is from July 1, 2007, to June 30, 2009, which is the financial crisis period.

We use the in-sample data to estimate an AR(1)-GARCH(1,1) model with student's t innovations. After getting the estimates for u_t , $h_t(\alpha)$, $H_t(\alpha)$ using the out-of-sample data, we implement the conditional backtests for VaR and ES using the new `dbptest` command.

Without Estimation Effects.—

Here we carry out the automatic portmanteau test (5) without considering the estimation effects, i.e. $\tau_j = 1$.

```
. dbptest H, mu(0.05)
```

```
Automatic Portmanteau test for serial correlation
```

```
Variable: H
```

Portmanteau (Q) statistic	=	2.8417
Prob > chi2(1)	=	0.0918
The number of lag(s) (from 1 to 23)	=	1

The displayed results are for cumulative violations at 10% risk level, i.e. $H_t(0.1)$. Under the correct model specification, we have $E[H_t(\alpha)] = \alpha/2$, so we set `mu` to be 0.05. We get an AQ statistic of 2.8417 and a P-value of 0.0918. Hence, the ES model is rejected at 10% significance level. It also reports the number of lag(s) chosen, which is 1 in this case.

Likewise, we carry out the conditional backtest for VaR using $h_t(\alpha)$. Following the rule-of-thumb that the coverage level for ES is twice (or approximately twice) that of VaR , We examine the autocorrelations of $h_t(0.05)$.

```
. dbptest h,mu(0.05)
```

```
Automatic Portmanteau test for serial correlation
```

```
Variable: h
```

Portmanteau (Q) statistic	=	0.7972
Prob > chi2(1)	=	0.3719
The number of lag(s) (from 1 to 23)	=	1

We now get an AQ statistic of 0.7972 and a P-value of 0.3719, we fail to reject the VaR model.

With Estimation Effects.—

To take the estimation effects into account, we use the command `rtau` to estimate τ_j first before we run the `dbptest` command.

```
. rtau lret, nlags(15) seriestype(es) cl(0.1) nobs(2658)
```

Asymptotic Variances of Autocorrelations

Order	Tau for ES
1	1.0027636
2	1.0192228
3	1.0192343
4	1.004399
5	1.0030891
6	1.0021455
7	1.0137747
8	1.0016341
9	1.0094143
10	1.0012676
11	1.0011319
12	1.0080588
13	1.0077699
14	1.0033674
15	1.0017961

```
. mat Tau_ES = e(tau)
```

```
. dbptest H, mu(0.05) tauvector(Tau_ES)
```

Automatic Portmanteau test for serial correlation

Variable: H

Portmanteau (Q) statistic	=	2.8338
Prob > chi2(1)	=	0.0923
The number of lag(s) (from 1 to 15)	=	1

```
. rtau lret, nlags(15) seriestype(var) cl(0.05) nobs(2658)
```

Asymptotic Variances of Autocorrelations

Order	Tau for VaR
-------	-------------

1	1.01014
2	1.0029985
3	1.0023986
4	1.0023737
5	1.0027832
6	1.0021056
7	1.001556
8	1.0014201
9	1.0011457
10	1.0009844
11	1.001431
12	1.0013224
13	1.0013889
14	1.0009824
15	1.0011676

```
. mat Tau_VaR = e(tau)

. dbptest h, mu(0.05) tauvector(Tau_VaR)

Automatic Portmanteau test for serial correlation
```

Variable: h

Portmanteau (Q) statistic	=	0.7892
Prob > chi2(1)	=	0.3743
The number of lag(s) (from 1 to 15)	=	1

Notice that the in-sample size here is 2658. The *AQ* test statistics for *ES* and *VaR* here are slightly lower than those without estimation effects. The tests conclusions remain the same, although the P-values are slightly bigger than before.

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