CAEPR Working Paper
#2013-002

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March, 2013

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Centralization or Decentralization in Multi-Agency Contracting Games?^{*}

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November 14, 2012

Abstract

This paper examines the centralized mechanism design procedure versus the decentralized menu design procedure in generalized pure-strategy multi-agency contracting games with both adverse selection and moral hazard. Ex post mechanisms are compared with ex post menus. Although mechanism design and menu design are strategically equivalent in single-agency situations, my main finding is that ex post menu design will be sub-optimal in multi-agency situations, if the associated transaction costs allow it. Ex post menu design is merely strategically equivalent to individual-based ex post mechanism design. Joint-based ex post mechanism design makes the principal better off than both of them. I also provide conditions under which the former strongly dominates the latter two and conditions under which they are all equivalent. I also apply the results to the study of incentive-based financial regulation with multiple banks, and I discuss why the financial regulatory contracting cannot be decentralized without loss of generality.

Keywords: multi-agency, ex post equilibrium, mechanism design, menu design, incentive compatibility, revelation principle, delegation principle

JEL Classification: C79 D82 D86

^{*}For their helpful comments and discussion, I gratefully acknowledge Frank Page, Seungjin Han, Phil Reny, Kim-Sau Chung, Yongchao Zhang, Sascha Baghestanian, Ronald M. Giammarino, Michael Koss, Robert Becker, and the participants/audiences at PET annual meeting 2012, Indiana University Microeconomics Workshop Fall 2012, and St Louis Fed/Missouri Economics Conference 2012. I am solely responsible for any errors.

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1 Introduction

This paper is about economic design in one-shot pure-strategy multi-agency contracting games with both adverse selection and moral hazard. Multi-agency denotes that a single principal contracts with multiple agents. To resolve the adverse selection and moral hazard problems, the principal needs to find a way to link a profile of outcome-contingent contracts and action-suggestions for the agents with each profile of their private information, so as to create incentives for the agents to enter into a contract-action selection profile in her¹ best interest. There are two major ways, or contracting procedures, to implement that. The first way is the mechanism design procedure. It requires the agents to report some messages to the principal. The principal first announces a communication mechanism associating contract-action pair profiles with report profiles. The second way is the menu² design procedure. The principal proposes a collection of contract profiles, namely a joint menu, to the agents. None of the agents need to report anything. They are entitled to choose directly a contract within the pre-offered menu and an action in the known action set.

Specifying contracts and actions for the agents can be regarded as a key decision in multi-agency games. Milgrom and Roberts (1992 P.114) claim the following:

Think about a situation in which there is a set of individuals who have various decisions to make and actions to perform... A particular decision is then decentralized if it is left to the individuals alone to make. In contrast, a centralized decision is one that is made at a higher level and communicated to or imposed on the individuals.

In this spirit, mechanism design can be viewed as a centralized contracting procedure, whereas menu design can be viewed as a decentralized contracting procedure. In essence, they are distinct contracting procedures. Many researchers study the menu design problem as a different self-contained procedure.³ They all treat a menu as a subset of the feasible contract set, and the contracting procedure is decentralized. Because menu design is more straightforward and can skip information communication in specifying contracts and actions, people have been interested in whether the contract-action choice can be delegated to the agents, that is, whether centralized mechanism design and decentralized menu design are strategically equivalent.

Previous studies demonstrate that no loss of generality is imposed by delegation of the contract-action choice in the single-agent situations. This fact is called the delegation principle. It indicates the strategic equivalence between mechanism design and menu design. In detail, if there is an optimal mechanism solving the mechanism design problem, there exists an optimal menu solving the menu design problem, vice versa, and solving these two problems will bring

¹Throughout the paper, masculine pronouns refer to the agents, whereas feminine pronouns apply to the principal.

²Some authors also use the term ‘‘catalogs’’ instead.

³See Martimort and Stole (2002), Page(1992), Page and Monteiro (2003), Carmona and Fajardo(2007), etc.

the same (expected) payoff to the principal in both cases. Page (1992) and Carlier (2001) characterize the incentive compatible direct mechanisms for single-principal single-agent contracting problems via the sets of contracts. Peters (2001), Martimort and Stole (2002), and Page and Monteiro (2003) study the strategic equivalence of menu design and mechanism design in multi-principal single-agent contracting problems. Is menu design, however, strategically equivalent to mechanism design in any case? What if there are multiple agents? After all, these two procedures have distinct implications and seemingly different game rules.

Compared with the single-agent environment, the multi-agency environment implies that the impacts of different agents' asymmetric information on the principal's objective will be interrelated. First, the agents are allowed to be not only heterogeneous but also fully interdependent. Each agent has a heterogeneous payoff normally depending on not only his own type, contract specification, and action but also those of other agents. Second, the principal's payoff will jointly depend on all the agents' types, contract specifications, and actions. One may suspect that the difference of mechanism design and menu design will be magnified in multi-agency environments with such interrelated impacts. In the general multi-agency situation, centralized mechanism design seems more pervasive in practice and in research. Thus, are these two major procedures no longer equivalent in this situation? Which one will be superior under what conditions?

This paper aims to investigate the comparison between centralized mechanism design and decentralized menu design in the general multi-agency environment. Multi-agency situations are frequently observed in principal-agent relationships. The government will allocate public goods and impose taxes on individuals. A company needs to compensate its executives. One monopolistic firm may perform procurement with several upstream firms. The essence of the multi-agency contracting game is a two-stage sequential game. The principal's strategy is to design optimal mechanisms or menus for the agents. Each pre-offered mechanism or joint menu actually defines a non-cooperative subgame for all the agents to play simultaneously. When the principal designs optimal mechanisms or joint menus, she must take into account what mechanism or joint menu may yield what particular equilibrium for the agents.

In this paper, ex post equilibria are of interest as the solution concepts of the subgame played by the agents in contracting games. In an incomplete information game, ex post (Nash) equilibrium (EPE) is the strategy profile under which every action profile is a Nash equilibrium at all type profiles. EPE has the ex post no-regret property, as no player would change his ex post equilibrium strategy even if he were to know the true type profile. It is weaker than dominant strategy equilibrium but stronger than Bayesian Nash equilibrium. The Wilson Doctrine (1987) states that a sound theory of games should not rely too heavily on assuming some features, such as one agent's probability assessment about another's preference or information, to be common knowledge. According to Chung and Ely (2007), detail freeness is the usual interpretation of the Wilson Doctrine. It means that the rules would not have to be tailored to any fine details of the environment in which it is employed.¹¹ EPE is one kind of solution concept that is detail free. Hence EPE is a more practical and robust solution concept. It has been increasingly studied in

mechanism design theory.⁴

This paper adopts this thought of detail free treatment. In mechanism design, the principal seeks an optimal mechanism from a class of pre-offered mechanisms inducing a subgame played by the agents in which some particular agents' participation strategy profile will be achieved as EPE. Such a class of mechanisms are called ex post mechanisms. In menu design, the principal seeks an optimal joint menu from a class of pre-offered joint menus inducing a subgame for all the agents in which some particular contract-action selection strategy profile of the agents will be achieved as EPE. Such class of joint menus are called ex post (joint) menus. In this respect, the contracting games over either ex post mechanisms or ex post menus are referred to as contracting games with ex post implementation.

My analysis also permits a certain feasible constraint on contracts for all the agents. This is consistent with many realistic situations, such as auctions, budget constraint, etc. Especially in menu design, it implies that the strategy set available for each agent may vary with the actions of the other agents in the subgame induced by a pre-offered joint menu. A subgame like this is actually a "constrained game," which is a generalized form of conventional games.⁵ The systematic description of "constrained games" in the normal form with complete information appears in the work of Rosen (1965). Ponstein (1966) uses the Glicksberg-Fan extension of Kakutani's theorem to derive the existence of Nash equilibrium points of constrained games. Since EPE is the strategy profile under which every action profile is just a Nash equilibrium at every type profile, one can still connect existing studies on Nash equilibria of "constrained games" to this context.

Hammond (1979) first discusses decentralization in multi-agency games with pure adverse selection and private valuations. To characterize the straightforward (dominant strategy) incentive compatible direct mechanisms, he suggests that a direct mechanism in the exchange economy can be decentralizable by some allocation sets parameterized by type profiles. Each agent can choose a contract from the set in his own interest. However, decentralization or delegation in this paper is centered on such self-contained menu design games. In the vein of decentralized procedure, the menus should be the sets of contracts independent of any parameters in self-contained menu design games.

Moreover, Han (2006) constructs a multi-principal multi-agent "bilateral contracting" environment with pure adverse selection and private valuations. He shows that "bilateral" mechanism design can be decentralized to menu design under Perfect Bayesian Equilibrium of randomized strategy "bilateral contracting" games. In his analysis, contract sets for individual agents are independent. The sets of messages that individual agent can report to individual principal are homogeneous and identical. All "bilateral" mechanisms the principals can offer to each agent are just the functions from the single constant message set to the independent set of contracts

⁴ See Dasgupta and Maskin (2000), Perry and Reny (2002), Bergemann and Valimaki (2002), Bergemann and Morris (2005, 2008, 2009, 2011), Chung and Ely (2006), and Bikhchandani (2006) among many others.

⁵ Constrained games are also called generalized games, shared constraint games, games with coupled constraints, or abstract economies.

available to that agent. A menu for each agent is regarded as a particular mechanism identifying whether some contract belongs to some closed subset of the independent set of contracts for the agent. Yet the focus of this paper is extended to be situations more general than the bilateral contracting environment in multi-agency games. The agents can be fully interdependent, and feasible contract sets for individual agents can be cross-constrained and heterogeneous. The report sets for different agents can be heterogeneous or unidentical. The general mechanisms are allowed to specify contracts and action suggestions for each agent jointly based on all the agents' reports. This is also consistent with the standard setting in conventional mechanism design theory and many practices. In addition, the self-contained menu design games are still addressed. Lastly, this paper centers on ex post implementation instead of Bayesian implementation.

My main finding in this paper is that ex post menu design will be sub-optimal relative to ex post mechanism design in multi-agency situations, if the associated transaction costs allow it. In this respect, there is a loss of generality when restricting attention to decentralized menu design. The revelation principle allows people to restrict attention to ex post incentive compatible (EPIC) direct mechanisms. The relevant delegation principle just indicates that ex post menus are only strategically equivalent to individual-based EPIC direct mechanisms, which associate each contract and action suggestion for an individual agent merely with his individual report. This extends the spirit of Han's menu theorem in the bilateral contracting environment, although EPE is addressed. Furthermore, joint-based EPIC direct mechanisms, which associate each contract and action suggestion for an individual agent with the joint report of all the agents, makes the principal better off than individual-based EPIC direct mechanisms and also ex post menus. The former can make the principal strictly better off than the latter two in some cases, e.g., when the optimal individual-based mechanism is strictly EPIC and can be modified with a small change to be a strict joint-based mechanism that makes the principal strictly better off. Meanwhile, if the impacts of different agents' asymmetric information on the principal's objective are independent and separate, those three ways of design are equivalent. The equivalence also occurs if feasible mechanisms are restricted to be individual-based. The main results of this paper endorse the rationale of using joint-base mechanisms and centralization to adopt richer informational reference to deal with the multi-agency information asymmetry problem.

The results of this paper are based on a generalized paradigm. Myerson (1982) has formulated a generalized Bayesian model setup of the principal-agent problem with both adverse selection and moral hazard in multi-agency cases. Based on topological structure and Borel measurability, Kadan, Reny and Swinkels (2011) also present a general formulation of the direct mechanism design problem with both adverse selection and moral hazard in single-principal single-agent cases. Following their paradigm, the model setting of this paper basically relies on general topological or metric structure and Borel measurability, so it is applicable in many particular circumstances.

My results are only of interest if there are economically interesting environments in which

EPIC mechanisms or ex post menus exist and optimal solutions to all the relevant contracting problems exist. In addition, it may be more meaningful to care about the existence of nontrivial EPIC mechanisms or ex post menus. A nontrivial EPIC mechanism denotes a nonconstant EPIC mechanism. Accordingly, a nontrivial ex post menu denotes an ex post menu that has at least two elements. Since EPE is still a strong solution concept, it is a moot point to ensure all the aforementioned existence in a general setting. In particular, Jehiel et al. (2006) shows that nontrivial ex post implementation will not be possible in generic joint-based mechanism design frameworks with pure adverse selection, multi-dimensional types, interdependent valuations, and quasilinear utility functions of the agents. One should be cautious in this point. On the other hand, since the set of individual-based ex post mechanisms is actually equivalent to a subset of joint-based EPIC mechanisms, the existence of (nontrivial) individual-based EPIC mechanisms or ex post menus is the sufficient but not necessary condition for the existence of (nontrivial) joint-based EPIC mechanisms. In this sense, centralization in terms of joint-based ex post mechanisms will always be the starting point for the principal to solve her contracting problems.

Section 2 presents basic elements of the model and provides some discussion and several examples within the model's scope. Section 3 explores the ex post mechanism design problem and proves the related revelation principle. Section 4 investigates the menu design problem and shows the delegation principle for multi-agency. The comparative analysis between all relevant contracting patterns is comprehensively delivered in Section 5. Section 6 examines an application of the main results in incentive-based financial regulation with multiple banks. I discuss why the financial regulatory contracting cannot be decentralized without loss of generality. Concluding remarks are given in section 7.

2 Basic Elements

There are one principal and n agents indexed by $i \in N = \{1, \dots, n\}$ in the multi-agency contracting game of interest. The principal will move first, and the agents will follow simultaneously and behave non-cooperatively. The principal needs the agents' participation to realize some economic objective of hers. Yet the principal is uninformed about some characteristics on the agents' side.

To save words, throughout this paper, the symbol $B(X)$ is reserved for Borel σ -field of a certain space X and the symbol $M(X, Y)$ for the set of all $(B(X), B(Y))$ -measurable functions from one space X to another space Y .

2.1 Agent Types and Actions

Each agent has two kinds of characteristics that are observable only to himself: hidden type and hidden action. The former is exogenous, whereas the latter is endogenous. They represent two categories of asymmetric information problems: adverse selection and moral hazard.

Agent i 's type is $t_i \in T_i$. Write $(t_i)_{i \in N} \in T = \prod_{i=1}^n T_i$ and $t_{-i} = (t_j)_{j \in N \setminus \{i\}} \in T_{-i} = \prod_{j \neq i} T_j$.

[Assumption 1] For each i , T_i is a Borel space, i.e. a Borel subset of a Polish space⁶.

μ_i is a probability measure defined on $B(T_i)$. μ is a probability measure on the associated product Borel σ -field $B(T)$. Thus, T and T_{-i} are also Borel spaces. $(T, B(T), \mu)$ is a probability measure space characterizing the common prior over the agents' private types.

Example 1 If the types of different agents are independent, $\mu = \prod_{i=1}^n \mu_i$, i.e. μ is the product probability measure on $B(T)$.

Example 1 T can be multi-dimensional. For a salesperson i , $T_i = \{\text{sales ability, sales environment, characteristics of firm, attitude}\} \subset [0, 1] \times [0, 1] \times \{\text{good, medium, bad}\} \times R$.

Example 2 There may exist correlated types. For instance, a public project is to be conducted in a community. Each resident i has a private valuation $v_i \in R$ for it. If the residents know some results of relevant environmental or geological tests and the results of the tests of various residents are correlated, then they will have a priori some information about each other's willingness to pay for the project. In other words, their types are correlated.

Agent i 's action is $a_i \in A_i$. Write $a = (a_i)_{i \in N} \in A = \prod_{i=1}^n A_i$ and $a_{-i} = (a_j)_{j \in N \setminus \{i\}} \in A_{-i} = \prod_{j \neq i} A_j$.

[Assumption 2] For each i , A_i is a compact metric space with Borel σ -field $B(A_i)$ of A_i . $B(A)$ is the associated product Borel σ -field of A . A_i contains an element a_i^0 , which denotes "don't participate".

Thus, A and A_{-i} are also compact metric spaces. Since a_i^0 is introduced, some agent with a certain type is permitted to voluntarily abstain from contracting. a_i^0 is commonly observable. All actions but a_i^0 chosen by agent i are observable only to i .

Example A_i can be multi-dimensional also. In Holmstrom and Milgrom (1991)'s study on multi-task principal-agent analysis, teachers' tasks can be separated to three categories: teaching basic skills, teaching higher-level thinking, and teaching other things. A teacher will exert efforts on these three tasks. They also suggest that one can model the agent's efforts across a continuum of tasks indexed by $\theta \in [0, 1]$. a_θ denotes the efforts the agent devotes to task θ . In this case, the agent's action space can even be infinite-dimensional.

⁶A complete separable metric space is called Polish space.

2.2 Outcomes

The end-of-period commonly observable outcome⁷ is $w \in \Omega$. The outcomes may include not only monetary outcomes but also some non-monetary outcomes. If the outcomes can be separately observed across the agents, let $w = (w_1, \dots, w_N) \in \Omega = \Omega_1 \times \dots \times \Omega_N$, where $w_i \in \Omega_i$ is the observable outcomes associated to agent i .

[Assumption 3] Ω is a closed metric space.

$\Phi \in \mathcal{P}(\Omega)$ is a probability measure defined on $(\Omega, \mathcal{B}(\Omega))$. So $(\Omega, \mathcal{B}(\Omega), \Phi)$ is a probability measure space. $\mathcal{P}(\Omega)$ is equipped with the metrizable topology of weak convergence. $\mathcal{P}(\Omega)$ has Borel σ -field $\mathcal{B}(\mathcal{P}(\Omega))$.

Given $(a, \theta) \in A \times T$, $p(a, \theta)$ is a probability measure defined on the space $(\Omega, \mathcal{B}(\Omega))$. It represents the common belief concerning how actions and types of the agents stochastically determine observable outcomes.

Denote the set of such probability measures by

$$\mathcal{P}(\Omega; A \times T) : \{p(a, \theta) : (a, \theta) \in A \times T\} \subset \mathcal{P}(\Omega)$$

[Assumption 4] For each closed subset E of Ω , $p(E, \theta)$ is continuous on $A \times T$. For each $(a, \theta) \in A \times T$, the probability measure $p(a, \theta)$ is absolutely continuous with respect to the probability measure Φ (denoted $\mathcal{P}(\Omega; A \times T) \ll \Phi$).

2. Contracts

The end-of-period commonly observable allocation for agent i is $d_i \in D_i$. Allocation d_i stipulates a certain relationship of transfer of economic interests or wealth between the principal and the agent i . It specifies gains from trade. Allocation d_i can be the reward from the principal to agent i in the context of employee compensation, a product-price pair in the context of nonlinear pricing, or a social alternative-transfer pair in the context of social choice. Write $d = (d_i)_{i \in N} \in D = \prod_{i=1}^N D_i$.

[Assumption 5] For each i , D_i is a metric space.

Thus, D is a metric space, too. The space $\mathcal{M}(\Omega, D_i)$ can be topologized with the topology of pointwise convergence.

All possible contracts available to agent i is $f_i \in K_i$. The set of feasible joint contracts is $K = \prod_{i=1}^N K_i$. Write $f = (f_i)_{i \in N} \in K$. K may contain some realistic constraints on the contract profiles the principal can offer to the agents. Let $f(w) = (f_i(w))_{i \in N}$ and $f_{-i}(w) = (f_j(w))_{j \in N \setminus \{i\}}$.

[Assumption 6] For each i , $K_i \subset \mathcal{M}(\Omega, D_i)$. K is a compact metric space.

There are many examples fitting these assumptions on contract sets in multi-agency situations.

⁷The observable outcome may be also called performance measurement, information system, or signals. Observability also implies verifiability throughout this paper.

Example 4 (1) Finite contract sets there are only finitely many contracts in each K_i . K is a compact metric space as any subset of $\bigcap_{i=1}^n K_i$.

() Product-price pairs with budget constraint each buyer i is offered a product-price pair (x_i, p_i) x_i is some product characteristics such as quantity quality etc. p_i is the price the seller can charge for i . This setting is frequently observed in nonlinear pricing games. So the joint feasible contract set can be either

$$K = \{(x_1, \dots, x_n, p_1, \dots, p_n) \in R \mid R : 0 \leq \sum_i p_i x_i \leq W\},$$

where $W \geq 0$ is the budget upper bound of the seller or

$$K = \{(x_1, \dots, x_n, p_1, \dots, p_n) \in R \mid R : 0 \leq p_i x_i \leq I_i, \text{ for each } i\},$$

where $I_i \geq 0$ is the budget upper bound of buyer i . In either case K is a compact metric space.

() Contract sets for a single object each bidder i is offered a pair (x_i, p_i) x_i is i 's payment to the seller. p_i is the probability that i gets the object. This setting is frequently observed in optimal auction games. So the joint feasible contract set can be

$$K = \{(x_1, \dots, x_n, p_1, \dots, p_n) \in R \mid R : \sum_i p_i \leq 1, p_i \geq 0, 0 \leq x_i \leq I_i, \text{ for each } i\},$$

where $I_i \geq 0$ is the wealth of bidder i . Obviously K is a compact metric space.

() Outcome-contingent contract sets assume that all the contracts are outcome-contingent. Then assume that for each i , (i) $K_i \subset M([I], R) \subset \bigcap_{i=1}^n K_i^s$ where each K_i^s is a sequentially compact subset of K_i for the topology of pointwise convergence on $[I]$, (ii) K_i^s contains no redundant contracts that is if for any two f_i and $\bar{f}_i$ in K_i^s satisfying $f_i(w) \leq \bar{f}_i(w)$ for some $w \in [I]$ and $\{w \in [I] : f_i(w) < \bar{f}_i(w)\} \neq \emptyset$ and (iii) K_i^s is uniformly bounded. By Proposition 1 in Gul & Stacchetti (1999) and the analysis in Page Monterio (2002) K_i^s is hence compact and metrizable for the topology of pointwise convergence. So is K .

2.4 Payoffs

Let $u : [I] \times D \times A \times T \rightarrow R$ denote the principal's utility function over outcomes, allocations, actions, and types with respect to all agents. $u_i : D \times A \times T \rightarrow R$ denotes agent i 's utility function defined over allocations, actions, and types with respect to all agents.

Given an agents' type profile θ , a contract profile f for the agents, and an agents' action profile a , the principal's outcome-expected payoff function $\mathcal{U} : K \times A \times T \rightarrow R$ is defined by

$$\mathcal{U}(f, a, \theta) = \int u(w, f(w), a, \theta) p(dw, a, \theta)$$

Given an agents' type profile θ , a contract profile f for the agents, and an agents' action profile a , the agent i 's outcome-expected payoff function $\mathcal{U}_i : K \times A \times T \rightarrow R$ is defined by

$$u_i(f, a) = \int_D u_i(f(w), a) p(dw|a)$$

[Assumption 7] u is continuous on $[0, 1] \times D \times A$ and Borel-measurable on T . For each i , u_i is continuous on $D \times A \times T$. $u(\cdot, f(\cdot), a, \cdot)$ is bounded for each $f \in K$, $a \in A$, and $\cdot \in T$. $u_i(f(\cdot), a, \cdot)$ is bounded for each $f \in K$, $a \in A$, and $\cdot \in T$.

This generic setting permits the agents to be fully interdependent.¹¹ In detail, there are externalities in contracting⁸ among the agents because each agent's payoff depends on not only his own contract but also the other agents' contracts. Correlated types and interdependent valuations⁹ are allowed instead of private independent valuations, because each agent's outcome-expected payoff is based on not only his own type but also the other agents' types. The synergies¹⁰ in agents' actions are also allowed since each agent's outcome-expected payoff is based on not only his own action but also the other agents' actions. In addition, the principal's expected payoff is permitted to depend jointly on all the agents' types, contracts, and actions. Hence, the agents will behave strategically, and the impacts of their respective asymmetric information will be interrelated.

Furthermore, there are several continuity and measurability properties of the outcome-expected payoff functions.

Proposition 1 (Continuity and measurability properties of outcome-expected payoff functions)

Suppose assumptions 7 hold. Then the following statements are true.

- (i) For each $i \in N$, $u_i(\cdot, \cdot, \cdot)$ is continuous on $K \times A \times T$.
- (ii) For each $\cdot \in T$, $u(\cdot, \cdot, \cdot)$ is continuous on $K \times A$ and for each $(f, a) \in K \times A$, $u(f, a, \cdot)$ is $B(T)$ -measurable.

Proof. see the Appendix. ■

Moreover, the σ -integrability of u is needed.

[Assumption 8] For all $(f, a) \in K \times A$, $u(f, a, \cdot)$ is σ -integrable on T .

2.5 Discussions and Examples

The basic model setup is intended to be as general as possible. The setup basically relies on general topological or metric structure and Borel measurability. The model does not entail order structure or linear algebraic structure. Although some natural orders or vector spaces may exist in many applications, no monotonicity or concavity of payoffs is assumed in the basic model setup. Differentiability or validity of First Order Approach is not necessarily required as well.

The model reduces to a pure moral hazard problem when T_i is a singleton for each i . It reduces to a pure adverse selection problem when for each i , A_i contains only two elements,

⁸Han (2006) also addresses contracting problems with externality in adverse selection environments.

⁹Inter-dependent valuations are also regarded as informational externalities. See Jehiel and Moldovanu (2001) among many others.

¹⁰The term "synergy" is from Edmans, Goldstein and Zhu (2011)

11participate11 and 11don't participate.11¹¹

The following examples address a few significant economic problems in real life. They satisfy the assumptions above.

Example 5 Pure moral hazard. As Holmstrom (1983) presents n workers engage in a team production. For each worker i , T_i is a singleton. Monetary reward $d_i \in D_i = [\underline{d}, \bar{d}]$. i 's effort $a_i \in A_i = [0, 1]$. The joint production set $[\underline{w}, \bar{w}]$. Output is generated by the parameterized probability $p(a)$ with an associated distribution $F(w|a)$. $F(a)$ is continuous in a . Then let $u(w, d, a, \theta) = w - \theta_i d_i$. Let $v_i(w, d, a, \theta) = v_i(d_i) - v_i(a_i)$ in which agent i 's utility is separable in money $v_i(\cdot)$ and cost of effort $v_i(\cdot)$. v_i and v_i are both continuous.

Example 6 Pure adverse selection. Consider n agents in the standard social choice mechanism design problem. Each i has his own private type $\theta_i \in T_i \subset \mathbb{R}$. $\theta(\cdot)$ denotes the common prior on the type profiles. Allocation $y = (x, s) \in Y \subset (X, S)$, where x is the agents consumption choice and s is the agents' transfer to the principal. Set $X = \prod_{i=1}^n [\underline{x}_i, \bar{x}_i]$ and $S = \prod_{i=1}^n [\underline{s}_i, \bar{s}_i]$. i 's action $a_i \in A_i = \{\text{participate}, \text{don't participate}\}$. i 's quasilinear payoff is $v_i(x, \theta_i, a_i, e) = v_i(x, e_i) - \theta_i$ where v_i is a concave function. The virtual social planner's utility may be $u(x, \theta, a, e) = \sum_{i=1}^n v_i(x, \theta_i, a_i, e)$.

Example 7 Moral hazard and adverse selection. A general contractor hires n subcontractors to perform a project. Each subcontractor i has his own private cost parameter $\theta_i \in T_i \subset \mathbb{R}$. $\theta(\cdot)$ denotes the common prior on type profiles $(\theta_i)_{i \in N}$. i 's effort is $a_i \in A_i = [0, 1]$. Monetary reward for each i is $d_i \in D_i = [\underline{d}, \bar{d}]$. Their performance (outcomes) can be measured separately. i 's performance is $w_i \in [l_i, \bar{w}_i]$. Joint performance is $w = (w_i)_{i \in N} \in [l, \bar{w}]$. $p(a, \theta)$ is the associated parameterized probability distribution over $[l, \bar{w}]$. $p(a, \theta)$ is continuous in a and θ . Then i 's payoff $v_i(w, d, a, \theta) = v_i(d_i) - v_i(a_i, \theta_i)$ is separable in money $v_i(\cdot)$ and cost of effort $v_i(\cdot)$. v_i and v_i are both continuous. The general contractor's payoff $u(w, d, a, \theta) = \sum_{i=1}^n w_i - \sum_{i=1}^n d_i$.

Mechanism Design Problem and the Revelation Principle

To deal with asymmetric information and attain second-best solutions, one classic way is mechanism design. This is a centralized procedure of contract selection. The principal needs to process decentralized information and have the agents report some message to her. Thus, she can design a mechanism, as a communication system, to map the messages to specific contracts and action recommendations to the agents. This analysis will basically follow the terminology and paradigm in the work of Myerson (1982) and Kadan et al. (2011).

¹¹ In some cases of pure adverse selection, e.g. some social choice scenarios, there are no outside options for the agents, or the agents always prefer 11participate11 to 11don't participate.11 So A_i can even be reduced to contain only one element, 11participate.11

.1 General Mechanisms and Direct Mechanisms

Agent i 's report to the principal is $r_i \in R_i$. The message that the principal sends to agent i is $m_i \in M_i$. Let $r = (r_i)_{i \in N} \in R = \prod_{i=1}^n R_i$ and $m = (m_i)_{i \in N} \in M = \prod_{i=1}^n M_i$. Let $k_i : R \rightarrow K_i$ (resp. $\bar{m}_i : R \rightarrow M_i$) specify a contract in the universal contract set (resp. a message) to agent i for each report profile of all agents. Let $\bar{k}_i : R_i \rightarrow K_i$ (resp. $\bar{m}_i : R_i \rightarrow M_i$) specify a contract in the universal contract set (resp. a message) to agent i for each report of single agent i .

[Assumption 9] For each $i \in N$, R_i and M_i are Borel spaces. For each i ,

$$k_i \in M(R, K_i), m_i \in M(R, M_i), \bar{k}_i \in M(R_i, K_i), \bar{m}_i \in M(R_i, M_i)$$

Definition 1 A joint-based general mechanism is a pair of functions

$$(k, m) \in F(R, K, M),$$

where $k = (k_i, k) : R \rightarrow K$ satisfying $(k_i(r), k(r)) \in K$ for each $r \in R$,
and $m = (m_i, m) : R \rightarrow M$

An individual-based general mechanism is a pair of functions

$$(k, \bar{m}) \in F(R, K, M),$$

where $k = (\bar{k}_i, \bar{k}) : R \rightarrow K$ satisfying $(\bar{k}_i(r_i), \bar{k}(r)) \in K$ for each $r \in R$,
and $\bar{m} = (\bar{m}_i, \bar{m}) : R \rightarrow M$

Definition 2 A report function for agent i is a function $p_i : T_i \rightarrow R_i$ specifying agent i 's report given each type of i . A final decision function for agent i is a function $\phi_i : M_i \times T_i \rightarrow A_i$ specifying the final action that agent i takes after i learns his type θ_i and receives some message m_i . Any pair (p_i, ϕ_i) is referred to as a participation strategy for i .

Thus, each (k, m) in $F(R, K, M)$ is still Borel measurable. So is each $(\bar{k}, \bar{m})$ in $\bar{F}_{JC}(R, K, M)$. Thus, the agent i 's participation strategy is $(p_i, \phi_i) \in \mathcal{P}_i$. Write

$$p = (p_i)_{i \in N} \in \mathcal{P} = \prod_{i=1}^n \mathcal{P}_i, \quad (p_i)_{i \in N} \in \prod_{i=1}^n \mathcal{P}_i$$

$$p_{-i} = (p_j)_{j \in N \setminus \{i\}} \in \mathcal{P}_{-i} = \prod_{j=1, j \neq i}^n \mathcal{P}_j, \quad (p_j)_{j \in N \setminus \{i\}} \in \prod_{j=1, j \neq i}^n \mathcal{P}_j,$$

$$p(\cdot) = (p_i(\cdot))_{i \in N}, p_{-i}(\cdot) = (p_j(\cdot))_{j \in N \setminus \{i\}}, (m, \cdot) = (\bar{m}_i(m_i, \cdot))_{i \in N}$$

[Assumption 10] For each $i \in N$, $\mathcal{P}_i \in M(T_i, R_i)$, $\mathcal{P}_i \in M(M_i \times T_i, A_i)$

After the joint-based (resp. individual-based) mechanism (k, m) (resp. $(\bar{k}, \bar{m})$) is offered, each agent i with type θ_i sends a report $p_i(\theta_i)$ to the principal. Under the joint-based (resp.

individual-based) mechanism, the principal is then committed to offering a contract $k_i(p(\cdot))$ (resp. $\bar{k}_i(p_i(\cdot))$) and sending a message $m_i(p(\cdot))$ (resp. $\bar{m}_i(p_i(\cdot))$) to each i . Next, i with type θ_i will take an action $a_i(m_i(p(\cdot)), \theta_i)$ (resp. $a_i(\bar{m}_i(p_i(\cdot)), \theta_i)$).

People are highly interested in a particular type of general mechanism with $R_i = T_i$ and $M_i = A_i$ for each i when the rest of the setting remains unchanged. Such mechanisms are called direct mechanisms. In addition, m (resp. $\bar{m}$) will be replaced with a (resp. $\bar{a}$) for distinction in direct mechanisms.

.2 Principal-agent Problem in the Contracting Game over EE Post Mechanisms

There are five stages in the principal-agent contracting game over mechanisms:

At Stage 1, the principal proposes a mechanism, which is commonly observable, to the agents.

At Stage 2, each agent unilaterally learns his true type, and the agents simultaneously send reports to the principal.

At Stage 3, through the pre-offered mechanism, the principal assigns contracts and sends messages to the agents according to their reports.

At Stage 4, after receiving the respective contracts and messages, the agents simultaneously decide to stay or leave and choose unobservable actions if participating.

At Stage 5, outcomes are realized and the contracts are implemented.

Each mechanism offered by the principal induces a simultaneous-moved subgame of the agents in which EPE is of interest as the solution concept. Some notations are further simplified as below:

$$\begin{aligned} k(p(\cdot)) &= (k_i(p(\cdot)))_{i \in N}, m(p(\cdot)) = (m_i(p(\cdot)))_{i \in N}, m_{-i}(p(\cdot)) = (m_j(p(\cdot)))_{j \in N \setminus \{i\}}, \\ \bar{k}(p(\cdot)) &= (\bar{k}_i(p_i(\cdot)))_{i \in N}, \bar{k}_{-i}(p_{-i}(\cdot)) = (\bar{k}_j(p_j(\cdot)))_{j \in N \setminus \{i\}}, \\ \bar{m}(p(\cdot)) &= (\bar{m}_i(p_i(\cdot)))_{i \in N}, \bar{m}_{-i}(p_{-i}(\cdot)) = (\bar{m}_j(p_j(\cdot)))_{j \in N \setminus \{i\}}, \\ (m(p(\cdot)), \cdot) &= (a_i(m_i(p(\cdot)), \theta_i))_{i \in N}, a_{-i}(m_{-i}(p_{-i}(\cdot)), \theta_{-i}) = (a_j(m_j(p(\cdot)), \theta_j))_{j \in N \setminus \{i\}}, \\ (\bar{m}(p(\cdot)), \cdot) &= (a_i(\bar{m}_i(p_i(\cdot)), \theta_i))_{i \in N}, a_{-i}(\bar{m}_{-i}(p_{-i}(\cdot)), \theta_{-i}) = (a_j(\bar{m}_j(p_j(\cdot)), \theta_j))_{j \in N \setminus \{i\}} \end{aligned}$$

There are several definitions related to the ex post (Nash) equilibrium of the subgame induced by the mechanism.

Definition The agents' participation strategy profile $(p, \cdot)$ is said to be an Ex Post Equilibrium (EPE) under a joint-based general mechanism (k, m) if for each $i \in N$ and each $\theta_i \in T_i$,

$$\begin{aligned} &\Diamond_i(k(p(\cdot)), (m(p(\cdot)), \cdot), \cdot) \\ 2 \quad &\Diamond_i(k(p_i(\cdot), p_{-i}(\cdot)), a_i(m_i(p_i(\cdot), p_{-i}(\cdot)), \theta_i), a_{-i}(m_{-i}(p_i(\cdot), p_{-i}(\cdot)), \theta_{-i}), \cdot), \cdot) \quad (1) \end{aligned}$$

for all $(p_i, i) \in \mathbf{r}_i$. Moreover such pair (k, m) is referred to as a joint-based ex post general Mechanism.

Definition 4 The agents' participation strategy profile (p, i) is said to be an Ex Post Equilibrium under an individual-based general contracting mechanism $(\bar{k}, \bar{m})$ if for each $i \in N$ and each $i \in T$,

$$\begin{aligned} & \forall_i(\bar{k}(p(i)), (\bar{m}(p(i)), i), i) \\ & 2 \quad \forall_i(\bar{k}_i(p_i(i)), \bar{k}_{-i}(p_{-i}(-i)), i(\bar{m}_i(p_i(i), i), -i(\bar{m}_{-i}(p_{-i}(-i), -i), i), \end{aligned} \quad (2)$$

for all $(p_i, i) \in \mathbf{r}_i$. Moreover such a pair $(\bar{k}, \bar{m})$ is referred to as an individual-based ex post general mechanism.

Definition 5 A joint-based direct mechanism (k, a) is Ex Post Incentive Compatible (EPIC) if it induces truthful reporting and obedient acting as the EPE for all the agents i.e. for each $i \in N$ and each $i \in T$,

$$\begin{aligned} & \forall_i(k(i), a(i), i) \\ & 2 \quad \forall_i(k(i, -i), a_i, a_{-i}(i, -i), i), \end{aligned} \quad (3)$$

for all $a_i \in A_i, i \in T_i$

Definition 6 An individual-based direct contracting mechanism $(\bar{k}, \bar{a})$ is EPIC if it induces truthful reporting and obedient acting as the EPE for all the agents i.e. for each $i \in N$ and each $i \in T$,

$$\begin{aligned} & \forall_i(\bar{k}(i), \bar{a}(i), i) \\ & 2 \quad \forall_i(\bar{k}_i(i), \bar{k}_{-i}(-i), a_i, \bar{a}_{-i}(-i), i), \end{aligned} \quad (4)$$

for all $a_i \in A_i, i \in T_i$

Throughout this paper, when the term "EPIC mechanisms" is used, it actually refers to the EPIC direct mechanism. Moreover, in EPE, if a true type profile is given, either the participation strategy profile or truthful reporting and obedient acting can actually be viewed as a Nash equilibrium. Recall that EPE is the strategy profile that will always be a Nash equilibrium for all the true type profiles.

Thus, there are four principal's optimization problems to address contracting games over ex post mechanisms.

(P1) joint-based ex post general mechanism design problem

$$\begin{aligned} & \max_{(k,m) \in \text{EF}(R,J,C,M)} \max_{(p,c) \in \text{Ex}^{\diamond}_T} U(k(p), (m(p), \cdot), \cdot) (d) \\ & \text{s.t. } (p, \cdot) \text{ is the EPE under } (k, m). \end{aligned}$$

(P1) individual-based ex post general mechanism design problem

$$\begin{aligned} & \max_{(\bar{k}, \bar{m}) \in \text{EF}(R,J,C,M)} \max_{(p,c) \in \text{Ex}^{\diamond}_T} U(\bar{k}(p), (\bar{m}(p), \cdot), \cdot) (d) \\ & \text{s.t. } (p, \cdot) \text{ is the EPE under } (\bar{k}, \bar{m}) \end{aligned}$$

(P2) joint-based EPIC mechanism design problem

$$\begin{aligned} & \max_{(k,a) \in \text{EF}(T,J,C,A)} U(k(\cdot), a(\cdot), \cdot) (d) \\ & \text{s.t. } (k, a) \text{ is EP} \end{aligned}$$

(P2) individual-based EPIC mechanism design problem

$$\begin{aligned} & \max_{(\bar{k}, \bar{a}) \in \text{EF}(T,J,C,A)} U(\bar{k}(\cdot), \bar{a}(\cdot), \cdot) (d) \\ & \text{s.t. } (\bar{k}, \bar{a}) \text{ is EP} \end{aligned}$$

Remark 2 The principal may also consider maximizing her ex-post payoff given any type profile instead. But this will not bring structural change in the analysis of this paper.

It is worth noting that the subgame induced by the pre-offered general mechanism may yield multiple equilibria. To solve the mechanism design problem, the principal needs to select one particular equilibrium for tie-breaking. She may have sufficient bargaining power to designate a particular equilibrium for the agents to play. Or, she can recommend the agents to play a particular equilibrium. Due to the focal-point effect, the agents will follow such a recommendation.¹² The principal considers the particular EPE $(p, \cdot)$ that gives the principal the highest possible expected utility under each general mechanism. By choosing the optimal ex post general mechanism, the entire principal-agent contracting game can achieve an equilibrium. In direct mechanism design, the principal just focuses on truth-telling and obedient-acting as the particular EPE under each direct mechanism. But does it also lead to an equilibrium for the entire principal-agent game by choosing the optimal EPIC mechanisms? The revelation principle will provide the answer.

¹²Myerson (1988) also talks about multiple equilibria and equilibrium selection in mechanism design in a survey.

• The Revelation Principle for EE Post Mechanisms

In parallel with Myerson's (1982) synthesis of the celebrated revelation principle for generalized randomized strategy Bayesian games with adverse selection and moral hazard, the revelation principle in this study is provided for the class of pure strategy ex post mechanism games with both adverse selection and moral hazard. Its implication is that one can restrict attention to the EPIC mechanisms out of the ex post general mechanisms.

Theorem 1 (The revelation principle for ex post mechanisms)

Suppose assumptions $\mathcal{A}$ hold. Given any EPE (p, γ) of the subgame played by the agents under any joint-based ex post general mechanism $(k, m) \in F(R, K, M)$ in (P_1) there exists a joint-based EP mechanism $(\bar{k}, \bar{a}) \in F(T, K, A)$ in (P_2) in which the principal obtains the same expected payoff as in the EPE (p, γ) of the given ex post general mechanism (k, m) . Furthermore the optimal joint-based EP mechanism solving (P_2) is also optimal in the class of all joint-based ex post mechanisms.

These results also apply to individual-based ex post general mechanisms and individual-based EP mechanisms.

Proof. See the Appendix. ■

With consideration of the setup of the four mechanism design problems in section 3.2, this revelation principle also suggests that it actually gives the principal the highest possible expected payoff under each given EPIC mechanism for the principal to pick truth-telling and obedient acting as the EPE for the agents. By choosing the optimal EPIC mechanism, the entire principal-agent game also achieves an equilibrium. Anyway, the good news is that one can simply study the EPIC mechanism design problem instead of the ex post general mechanism design problem without loss of any generality.

4 Menu Design Problem and the Delegation Principle

4.1 Menus

An alternative way for the principal to seek a second best solution in contracting games is to design menus. This is a decentralized procedure of contract selection. The principal does not need to process decentralized information or have the agents send messages to specify contracts for the agents. Instead she can design a joint menu, i.e. a subset of the feasible joint contract set, for the agents. The agents will pick the contracts on their own accord from the joint menu.

The possible contract menu for agent i is $C_i \in P(K_i)$, where $P(K_i)$ is the power set of K_i . In view of feasible constraints in the joint contract set, the joint feasible menu is $C \in P_f(K) = \{(C_1, \dots, C_n) \mid (C_1, \dots, C_n) \in K\}$.

[Assumption 11] $P_f(K)$ is a collection of nonempty, closed subsets of K .

Because K is a compact metric space, $P_f(K)$ equipped with the Hausdorff metric is also a compact metric space (see Theorem 3.85 in Aliprantis and Border (2006)).

4.2 Principal-Agent Problem in the Contracting Game over EE Post Menus

There are three stages in the principal-agent contracting game over menus:

At Stage 1, the principal proposes a joint contract menu, which is commonly observable, to the agents.

At Stage 2, each agent unilaterally learns his true type. The agents simultaneously select the contracts from the pre-offered joint menu and decide to stay or leave and choose unobservable actions if participating.

At Stage 3, outcomes are realized and the contracts are implemented.

Each menu offered by the principal induces a simultaneous-moved subgame played by the agents. Under a joint menu C , each agent i 's strategy is a pair of functions $f_i : T_i \rightarrow K_i$ and a_i denotes i 's action selection according to his type.

[Assumption 12] For each $i \in N$, $f_i \in M(T_i, K_i)$, $a_i \in M(T_i, A_i)$

Let

$$f = (f_i)_{i \in N}, f(-i) = (f_i(-i))_{i \in N}, f_{-i}(-i) = (f_j(-j))_{j \in N \setminus \{i\}}, \\ a = (a_i)_{i \in N}, a_{-i}(-i) = (a_j(-j))_{j \in N \setminus \{i\}}$$

Note that this subgame is actually constrained by $f(-i) \in C$ for each $i \in N$. Let $F_C = \{f \text{ for } f(-i) \in C \text{ for each } i, a_i \in M(T_i, A_i)\}$.

Each f and a are also Borel measurable. EPE is still regarded as the solution concept of the subgame. EPE for this constrained game is defined to be consistent with the classic literature as below:

Definition 7 A contract-action selection profile $(f(-i), a(-i))$ is said to be an EPE under a joint menu C if for each $i \in N$ and each $t_i \in T_i$,

$$a_i(t_i), a_{-i}(-i), (f(-i), a_{-i}(-i)), \quad (5)$$

for all $f_i \in M(T_i, K_i)$ satisfying $(f_i(t_i), f_{-i}(-i)) \in C$ and all $a_i \in M(T_i, A_i)$. Moreover such a joint menu C is referred to as an ex post (joint) menu.

In EPE, if a true type profile is given, the realized contract-action pair is a Nash equilibrium of the (constrained) subgame defined by an ex post menu. The principal can deduce that the agents will have the EPE contract-action selection profile in the subgame. The agents will not regret even if they know the type profile ex post. The principal will have an optimization problem to address this contracting game over ex post menus.

(P) Ex post menu design problem:

$$\max_{C \in \mathcal{C}} \max_{\mathbf{f} \in F_C, \mathbf{a} \in A} U(\mathbf{f}, \mathbf{a}) \quad (d)$$

s.t. $(\mathbf{f}, \mathbf{a})$ is the EPE under C

Again, in view of multiple equilibria, for tie-breaking the principal may designate or recommend $(\mathbf{f}, \mathbf{a})$ in her best interest for the agents with type profile $\mathbf{r}$ to follow. In this way, the principal can link one contract-action pair with each type profile $\mathbf{r}$.

4. The Delegation Principle for Multi-Agency

Is there any connection between the menu design problem and the mechanism design problem? In fact one can completely characterize all ex post menus for contracting games using individual-based EPIC mechanisms. One can then show the strategic equivalence between the individual-based EPIC mechanism design problem and the ex post menu design problem. These are summarized by the delegation principle. In this sense, all individual-based EPIC mechanisms can be decentralized via ex post menus.

Consider the set-valued mapping $\langle I : T \rightarrow P_f(K) \rightarrow A$ defined by

$$\langle I(\mathbf{r}) \rangle \in C \quad A : (\mathbf{f}, \mathbf{a}) \text{ is the EPE under } C\}$$

It is deducible by the principal and represents the $\mathbf{r}$ -type-profile agents' joint ex post equilibrium response to any menu offer C .

Definition 8 Given $C \in P_f(K)$, $\langle I(\mathbf{r}, C) \rangle$ is said to be well-defined if for each $\mathbf{r}$, $\langle I(\mathbf{r}, C) \rangle$ is nonempty.

A well-defined $\langle I(\mathbf{r}, C) \rangle$ implies that the agents with each given type profile $\mathbf{r}$ can possess at least one Nash equilibrium contract-action pair under the joint menu C . In other words, there exists at least one ex post menu C . Moreover, the continuity of each U_i will imply that well-defined $\langle I(\mathbf{r}, C) \rangle$ has a closed graph. Hence, Lemma 17.51 (Aliprantis and Border 2006) implies that the corresponding set of Nash equilibria¹³ is a closed subset of $C \times A$ and then is compact. In other words, $\langle I(\mathbf{r}, C) \rangle$ is compact-valued. Moreover, Borel measurability of $\langle I(\mathbf{r}, C) \rangle$ can be guaranteed.

Lemma 1 Suppose that assumptions 7 and 2 hold. For any $C \in P_f(K)$ if $\langle I(\mathbf{r}, C) \rangle$ is well-defined $\langle I(\mathbf{r}, C) \rangle$ has a closed graph.

Proof. See the Appendix. ■

Proposition 2 Suppose that assumptions 7 and 2 hold. For any $C \in P_f(K)$ satisfying $\langle I(\mathbf{r}, C) \rangle$ is well-defined $\langle I(\mathbf{r}, C) \rangle$ is a Borel-measurable set valued function from T to $C \times A$

¹³ Even if the game is constrained, classic fixed point theorems related to Nash equilibria can still apply. One can refer to Rosen (1965) and Ponstein (1966).

Proof. See the Appendix. ■

The following theorem describes the complete characterization of all individual-based EPIC mechanisms via ex post menus.

Theorem 2 (The delegation principle version 1)

Suppose that assumptions 7, and 2 hold. Given a contracting mechanism $(\bar{k}, \bar{a}) \in \bar{F}(T, K, A)$ the following statements are true.

(i) If $(\bar{k}(\cdot), \bar{a}(\cdot))$ is EP then there exists a joint menu $C \in P_f(K)$ such that $\langle I(\cdot), C \rangle$ is well-defined and $(\bar{k}(\cdot), \bar{a}(\cdot))$ is a Borel-measurable selection from $\langle I(\cdot), C \rangle$ that is $(\bar{k}(\cdot), \bar{a}(\cdot)) \in \langle I(\cdot), C \rangle$ for all $\theta \in T$

(ii) If there exists a joint menu $C \in P_f(K)$ such that $\langle I(\cdot), C \rangle$ is well-defined and $(\bar{k}(\cdot), \bar{a}(\cdot))$ is a Borel-measurable selection from $\langle I(\cdot), C \rangle$, then $(\bar{k}(\cdot), \bar{a}(\cdot))$ is EP.

Proof. (i) Assume that $(\bar{k}(\cdot), \bar{a}(\cdot)) \in \bar{F}(T, K, A)$ is EPIC. Define

$$C = \bigcap_{i=1}^n \overline{I\{(\bar{k}_i(\theta_i) : \theta_i \in T_i)\}} \times K, \text{ where } \overline{I} \text{ denotes the closure.}$$

First claim that $(\bar{k}(\cdot), \bar{a}(\cdot)) \in \langle I(\cdot), C \rangle$ for all $\theta \in T$, that is, for each $i = 1, \dots, n$, and each $\theta_i \in T_i$,

$$\varphi_i(\bar{k}(\cdot), \bar{a}(\cdot), \theta_i) \geq \varphi_i(f_i(\theta_i), \bar{k}_{-i}(\theta_{-i}), a_i(\theta_i), \bar{a}_{-i}(\theta_{-i}), \theta_i),$$

for all $f_i \in M(T_i, K_i)$ satisfying $(f_i(\theta_i), \bar{k}_{-i}(\theta_{-i})) \in C$ and all $a_i \in M(T_i, A_i)$

Suppose not. Then for some agent j , some agents' types profile (θ_j, θ_{-j}) , some $f_j \in M(T_j, K_j)$ satisfying $(f_j(\theta_j), \bar{k}_{-j}(\theta_{-j})) \in C$, and some $a_j \in M(T_j, A_j)$,

$$\varphi_j(\bar{k}(\cdot), \bar{a}(\cdot), \theta_j) < \varphi_j(f_j(\theta_j), \bar{k}_{-j}(\theta_{-j}), a_j(\theta_j), \bar{a}_{-j}(\theta_{-j}), \theta_j)$$

Because of the definition of C , any section of C is still closed. Thus, for any θ_{-j} , there exists a sequence of type $\{\theta_{j,l}\}_l$ in T_j such that $(\bar{k}_j(\theta_{j,l}), \bar{k}_{-j}(\theta_{-j})) \in C$, as $l \rightarrow \infty$

But since $\varphi_j(\bar{k}(\cdot), \bar{a}(\cdot), \theta_j) < \varphi_j(f_j(\theta_j), \bar{k}_{-j}(\theta_{-j}), a_j(\theta_j), \bar{a}_{-j}(\theta_{-j}), \theta_j)$, by the continuity of $\varphi_j(\cdot, \cdot, \cdot)$, the fact that $(\bar{k}_j(\theta_{j,l}), \bar{k}_{-j}(\theta_{-j})) \in C$, as $l \rightarrow \infty$, implies that for l large enough,

$$\varphi_j(\bar{k}(\cdot), \bar{a}(\cdot), \theta_j) < \varphi_j(\bar{k}_j(\theta_{j,l}), \bar{k}_{-j}(\theta_{-j}), a_j(\theta_j), \bar{a}_{-j}(\theta_{-j}), \theta_j)$$

This contradicts the fact that $(\bar{k}(\cdot), \bar{a}(\cdot))$ is EPIC.

Therefore, $(\bar{k}(\cdot), \bar{a}(\cdot))$ is a Borel-measurable selection from $\langle I(\cdot), C \rangle$. Clearly, $\langle I(\cdot), C \rangle$ is well-defined.

(ii) Assume that $(\bar{k}(\cdot), \bar{a}(\cdot)) \in \langle I(\cdot), C \rangle - C$ for all $\theta \in T$

For all $i \in N$, all $t \in T$, all $f_i \in M(T_i, K_i)$ satisfying $(f_i(t), \bar{k}_{-i}(t)) \in C$, and all $a_i \in M(T_i, A_i)$,

$$v_i(\bar{k}(t), \bar{a}(t)) \geq v_i(f_i(t), \bar{k}_{-i}(t), a_i(t), \bar{a}_{-i}(t)),$$

For each $j \in N$ and each $t \in T$, since there are some f_j satisfying $f_j(t) = \bar{k}_j(t)$ for any a_j satisfying $a_j(t) = \bar{a}_j(t)$ for any $a_j \in A_j$, we have

$$v_j(\bar{k}(t), \bar{a}(t)) \geq v_j(\bar{k}_j(t), \bar{k}_{-j}(t), a_j(t), \bar{a}_{-j}(t)),$$

for all $t \in T_j$ and all $a_j \in A_j$. Thus, $(\bar{k}(\cdot), \bar{a}(\cdot))$ is EPIC. ■

Furthermore, the ex post menu design problem (P) can be rewritten in a compact way as long as $\langle l(\cdot, C) \rangle$ is well-defined for some $C \in P_f(K)$:

$$\max_{C \in (JC)} \max_{(f(t), a(t)) \in \langle l(t, C) \rangle} v(f(t), a(t)) \quad (d)$$

Now define the feasible individual-based EPIC mechanism set

$$IC^I : \{(\bar{k}(\cdot), \bar{a}(\cdot)) \in \bar{F}(T, K, A) : (\bar{k}(\cdot), \bar{a}(\cdot)) \text{ is EPIC.}\}$$

The individual-based ex post direct mechanism game can also be stated compactly as

$$\max_{(\bar{k}, \bar{a}) \in IC^I} v(\bar{k}(\cdot), \bar{a}(\cdot)) \quad (d)$$

Besides, define the equivalent mechanism set induced by a joint feasible menu C

$$E_{\langle l \rangle}(C) : \{(\bar{k}(\cdot), \bar{a}(\cdot)) \in \bar{F}(T, K, A) : (\bar{k}(\cdot), \bar{a}(\cdot)) \in \langle l(\cdot, C) \rangle \text{ for all } t \in T\}$$

It denotes the set of all measurable selections from $\langle l(\cdot, C) \rangle$ in $\bar{F}(T, K, A)$ for a given menu $C \in P_f(K)$. Next, define the overall equivalent mechanism set induced by the joint feasible menu set

$$E_{\langle l \rangle} : \max_{C \in (JC)} E_{\langle l \rangle}(C).$$

The delegation principle version 1 implies that

$$IC^I \subseteq E_{\langle l \rangle}.$$

Let us first see an important lemma that is very useful in establishing the strategic equivalence of individual-based direct mechanism design problem and menu design problem.

Lemma 2 Suppose assumptions 7, and 2 hold. For each $C \in P_f(K)$ satisfying

$\langle I, C \rangle$ is well-defined there exists some $(\bar{k}, \bar{a}) \in E_{\langle I, C \rangle}$ such that

$$\hat{U}(\bar{k}(t), \bar{a}(t), t) = \max_{(f(t), a(t)) \in \langle I, C \rangle} \hat{U}(f(t), a(t), t),$$

for all $t \in T$. Moreover the function $\max_{(f,a) \in \langle I, C \rangle} \hat{U}(f, a, \cdot)$ is Borel measurable.

Proof. See the Appendix. ■

Now we are ready to see one main result on the strategic equivalence of the individual-based EPIC mechanisms design problem and the ex post menu design problem.

Theorem (The delegation principle version)

Suppose assumptions 1 and 2 hold. Then the following statements are true

(i) $f(\bar{k}(\cdot), \bar{a}(\cdot))$ solves the contracting problem over individual-based EP mechanisms given by

$$\max_{(\bar{k}, \bar{a}) \in IC^J} \hat{U}(\bar{k}(\cdot), \bar{a}(\cdot), \cdot)(d),$$

then $C = \bigcap_{i=1}^n I\{\bar{k}_i(\cdot) : \cdot \in T_i\} \cap K$ solves the contracting problem over ex post menus given by

$$\max_{C \in (JC)} \max_{(f(t), a(t)) \in \langle I, C \rangle} \hat{U}(f(t), a(t), t)(d)$$

(ii) $f \in C$ solves the contracting problem over ex post menus given by

$$\max_{C \in (JC)} \max_{(f(t), a(t)) \in \langle I, C \rangle} \hat{U}(f(t), a(t), t)(d),$$

then $(\bar{k}(\cdot), \bar{a}(\cdot)) \in E_{\langle I, C \rangle}$ satisfying

$$(\bar{k}(\cdot), \bar{a}(\cdot)) \in \arg \max_{(f(t), a(t)) \in \langle I, C^* \rangle} \hat{U}(f(t), a(t), t)$$

for all $t \in T$ solves the contracting problem over individual-based EP mechanisms given by

$$\max_{(\bar{k}, \bar{a}) \in IC^J} \hat{U}\left(\bigcap_{i=1}^n \bar{k}_i(\cdot), \bigcap_{i=1}^n \bar{a}_i(\cdot), \cdot\right)(d)$$

In both cases the optimal objective values of the two problems are equal.

Proof. (i) Since $(\bar{k}(\cdot), \bar{a}(\cdot)) \in \langle I, C \rangle$, we have

$$\max_{(f(t), a(t)) \in \langle I, C^* \rangle} \hat{U}(f(t), a(t), t)(d) \geq \hat{U}(\bar{k}(\cdot), \bar{a}(\cdot), \cdot)(d)$$

Thus, for all $(\bar{k}, \bar{a}) \in IC^I$,

$$\bigotimes_T (\bar{k}(\cdot), \bar{a}(\cdot), \cdot)(d) \geq \bigotimes_T (\bar{k}(\cdot), \bar{a}(\cdot), \cdot)(d)$$

Then, by the delegation principle version 1, $IC^I = \bigcup_{CE \in JC} E_{<I} = \bigcup_{CE \in JC} E_{<I}(C)$

Hence, for all $(\bar{k}, \bar{a}) \in \bigcup_{CE \in JC} E_{<I}(C)$, we have

$$\bigotimes_T (\bar{k}(\cdot), \bar{a}(\cdot), \cdot)(d) \geq \bigotimes_T (\bar{k}(\cdot), \bar{a}(\cdot), \cdot)(d) \quad (6)$$

Moreover, by Lemma 2, for each $C \in P_f(K)$, there exists some $(\bar{k}, \bar{a}) \in E_{<I}(C)$ such that $\bigotimes_T (\bar{k}(\cdot), \bar{a}(\cdot), \cdot) \geq \max_{(f(t), a(t)) \in E_{<I}(t, C)} \bigotimes_T (f(\cdot), a(\cdot), \cdot)$ for all $E \in T$

Thus, by (6), for each $C \in P_f(K)$, we have

$$\begin{aligned} & \bigotimes_T (\bar{k}(\cdot), \bar{a}(\cdot), \cdot)(d) \\ & \geq \max_{(f(t), a(t)) \in E_{<I}(t, C)} \bigotimes_T (f(\cdot), a(\cdot), \cdot)(d) \end{aligned}$$

Therefore, $\bigotimes_T \max_{(f, a) \in E_{<I}(t, C^*)} \bigotimes_T (f, a, \cdot)(d) \geq \bigotimes_T \max_{(f(t), a(t)) \in E_{<I}(t, C)} \bigotimes_T (f(\cdot), a(\cdot), \cdot)(d)$ for all $C \in P_f(K)$. Hence, C solves the given contracting game over menus.

Clearly,

$$\begin{aligned} & \max_{CE \in JC} \max_{(f(t), a(t)) \in E_{<I}(t, C)} \bigotimes_T (f(\cdot), a(\cdot), \cdot)(d) \\ & \geq \bigotimes_T (\bar{k}(\cdot), \bar{a}(\cdot), \cdot)(d) \geq \max_{(\bar{k}, \bar{a}) \in IC^I} \bigotimes_T (\bar{k}(\cdot), \bar{a}(\cdot), \cdot)(d) \end{aligned}$$

(ii) For each $C \in P_f(K)$, by hypotheses,

$$\begin{aligned} & \bigotimes_T (\bar{k}(\cdot), \bar{a}(\cdot), \cdot)(d) \geq \max_{(f(t), a(t)) \in E_{<I}(t, C^*)} \bigotimes_T (f(\cdot), a(\cdot), \cdot)(d) \\ & \geq \max_{(f(t), a(t)) \in E_{<I}(t, C)} \bigotimes_T (f(\cdot), a(\cdot), \cdot)(d) \geq \bigotimes_T (\bar{k}(\cdot), \bar{a}(\cdot), \cdot)(d), \end{aligned}$$

for all $(\bar{k}(\cdot), \bar{a}(\cdot)) \in E_{<I}(C)$ satisfying $\bigotimes_T (\bar{k}(\cdot), \bar{a}(\cdot), \cdot) \geq \max_{(f(t), a(t)) \in E_{<I}(t, C)} \bigotimes_T (f(\cdot), a(\cdot), \cdot)$ for any $E \in T$. The existence of such a $(\bar{k}(\cdot), \bar{a}(\cdot))$ is ensured by Lemma 2. It implies that

$$\bigotimes_T (\bar{k}(\cdot), \bar{a}(\cdot), \cdot)(d) \geq \max_{(\bar{k}, \bar{a}) \in E} \max_{: E_{<I}(C)} \bigotimes_T (\bar{k}(\cdot), \bar{a}(\cdot), \cdot)(d) \quad (7)$$

$$\max_{(k,a) \in IC^J - T} \Phi(k(\cdot), a(\cdot), \cdot) (d)$$

$$IC^J : \{(k(\cdot), a(\cdot)) \in F(T, K, A) : (k(\cdot), a(\cdot)) \text{ is EPIC.}\}$$

(P2) can be rewritten in a compact form as below:

$$\max_{(\bar{k}, \bar{a}) \in IC^S - T} \Phi(\bar{k}(\cdot), \bar{a}(\cdot), \cdot) (d)$$

$$IC^I : \{(\bar{k}(\cdot), \bar{a}(\cdot)) \in \bar{F}(T, K, A) : (\bar{k}(\cdot), \bar{a}(\cdot)) \text{ is EPIC.}\}$$

Some manipulation needs to be utilized to compare (P2) and (P2'). Let $\pi_i : T \rightarrow T_i$ be the projection function defined by $\pi_i(\mathbf{t}, \mathbf{a}) = t_i$. Define the projective joint-based direct mechanism set

$$F_{\pi}(T, K, A) : \{((\bar{k}_i \circ \pi_i)_{i \in N}, (\bar{a}_i \circ \pi_i)_{i \in N}) \in F(T, K, A) : (\bar{k}, \bar{a}) \in \bar{F}(T, K, A)\}$$

Thus, $F_{\pi}(T, K, A) = F(T, K, A)$. Then it is safe to define the projective joint-based EPIC mechanism set

$$IC^J : \{((\bar{k}_i \circ \pi_i)_{i \in N}, (\bar{a}_i \circ \pi_i)_{i \in N}) \in F(T, K, A) : (\bar{k}, \bar{a}) \in IC^I\}$$

Observe that IC^I and IC^J are equivalent, i.e. they are in 1-1 correspondence with each other.

Now consider a new mechanism design problem (P2'):

$$\max_{(k,a) \in IC^J - T} \Phi(k(\cdot), a(\cdot), \cdot) (d)$$

Obviously, (P2) and (P2') will achieve the identical maxima at optimal 1-1 corresponding optimal mechanisms $(\bar{k}, \bar{a})$ and $(k, a) = ((\bar{k}_i \circ \pi_i)_{i \in N}, (\bar{a}_i \circ \pi_i)_{i \in N})$. In this sense, one can view (P2) and (P2') as essentially equivalent problems. Thus one can convert (P2) to (P2') and alternatively compare (P2) with (P2') to check the welfare comparison between joint-based mechanism design and individual-based mechanism design.

Apparently, $IC^J = IC^I$. With the same objective function, (P2') will hence yield a solution at least as good as (P2) and also (P) due to the delegation principle. This result is summarized in Theorem 4.

Theorem 4 Assume that optimal solutions to (P2) and (P2') exist. When assumptions 1 and 2 hold joint-based EP mechanism design weakly dominates both individual-based EP mechanism design and ex post menu design.

Compared with individual-based EPIC mechanisms, joint-based EPIC mechanisms suggest that the principal's contract specification for each agent is on the basis of peer types (reports). She can refer to not only such agent's information communication but also all the other agents'. With a richer reference from information communication, the principal can enhance her efficiency of decision making to deal with information asymmetry. The revelation principle suggests that this result can be extended from EPIC mechanisms to general mechanisms. Hence, joint-based mechanism design is more frequently observed in practice and of more interest to researchers.

5.2 Strong Dominance of Joint-based EE Post Mechanisms

When does the joint-based ex post mechanism design strongly dominate the individual-based ex post mechanism design and ex post menu design? Joint-based mechanism design strongly dominates individual-based mechanism design (resp. menu design) if the solution to the joint-based mechanism design problem yields higher expected payoff to the principal than the solution to the individual-based mechanism design problem (resp. menu design problem), i.e., joint-based mechanism design makes the principal strictly better off than individual-based mechanism design (resp. menu design). Due to the revelation principle, it suffices to study EPIC mechanisms solely.

Define the principal's expected payoff by using direct mechanisms

$$U(k, a) = \sum_T \theta(k(i), a(i), i)(d),$$

for each (k, a) . In general, $(P2)$ strongly dominates $(P2')$ and (P') if and only if there exists an element (k, a) in $\mathcal{IC}^J$ under which the principal can achieve a higher expected payoff than the optimal objective value of $(P2')$, given that all problems have optimal solutions. An immediate candidate of such (k, a) is some perturbation of the optimal individual-based EPIC mechanisms, which becomes a *pure* joint-based mechanism that preserves EPIC and brings higher expected payoff to the principal. It is tractable to find such (k, a) in finite contracting games, since there are only finitely many mechanisms. Yet things could be more complicated in infinite contracting games. Nevertheless, there is an intuitive sufficient condition of the strong dominance that can still work in infinite environments when the solutions to individual-based mechanism design problems are strict EPIC. Let us see the definition of Strict Ex Post incentive compatibility¹⁴ first.

Definition 9 A direct mechanism (k, a) is strict EPIC if for all $i, j, i \neq j$ and $a_i \neq a_j(i)$

$$v_i(k(i), a(i), i) > v_i(k(i, -i), a_i, a_{-i}(i, -i), i)$$

Proposition Suppose that Assumptions 1 and 2 hold and the optimal solutions to $(P2)$ and $(P2')$ are $(k, a) \in \mathcal{IC}^J \setminus \mathcal{IC}_I^J$ and $(k, a) \in \mathcal{IC}_I^J$ respectively. If

¹⁴This is an extension of the definition of strict EPIC give by Bergemann and Morris (2008, 2009, 2011).

- (i) (k^*, a^*) is strict EP
- (ii) there exists a sequence $\{(k^l, a^l)\}_l$ in $F(T, K, A) \setminus F_{tr}(T, K, A)$ such that as $l \rightarrow \infty$, $k^l \rightarrow k^*$, $a^l \rightarrow a^*$ for each i and for some N , $U(k^l, a^l) \geq U(k^*, a^*)$ whenever $l \geq N$, then joint-based EP mechanism design strongly dominates both individual-based EP mechanism design and ex post menu design.

Proof. Pick the sequence $\{(k^l, a^l)\}_l$ in $F(T, K, A) \setminus F_{tr}(T, K, A)$ as described in hypothesis (ii).

$\varphi_i(\cdot, \cdot)$ is continuous for all i and $\cdot$. Thus for some N_c , when $l \geq N_c$, the strict EPIC optimal solution to (P2) implies that

$$\varphi_i(k^l(i), a^l(i)) \geq \varphi_i(k^l(i), a_i, a_{-i}^l(i), -i),$$

for all i , $i \in I$ and $a_i = a_j^l(i)$

Clearly, $(k^l, a^l) \in C^J \setminus C_I^J$. Now pick $m = \max\{N, N_c\}$. $U(k^m, a^m) \geq U(k^*, a^*)$ by hypothesis (ii).

Furthermore, $U(k^*, a^*) \geq U(k^m, a^m) \geq U(k^*, a^*)$

Thus, (P2) makes the principal strictly better off than (P2) and (P2) as well as (P) by the delegation principle. ■

emark 1 It is not hard to ensure the hypothesis of strict EP (k^*, a^*) in some economically natural applications. For instance if for each i , $\varphi_i(f, a) = \varphi_i(f, a)$, either $\varphi_i(f, a, \cdot) \geq \varphi_i(f, a, \cdot)$ or $\varphi_i(f, a, \cdot) < \varphi_i(f, a, \cdot)$, then any (k^*, a^*) must be strict EP as long as it exists to be non-constant.

If the optimal solution to (P2) (k^*, a^*) can be tended by a sequence of ¹¹pure joint-based direct mechanisms that are not equivalent to individual-based direct mechanisms, there exists some element in the sequence that can inherit the EPIC property from the strict EPIC (k^*, a^*) . Therefore, (k^*, a^*) must be dominated at least by that element from the viewpoint of the principal. In addition, if the elements of the sequence will eventually bring higher expected payoff to the principal, (P2) strongly dominates (P2) and (P). Roughly speaking, the strong dominance exists if the optimal solution to (P2) is strict EPIC and can be modified with a small change to be a joint-based mechanism that makes the principal strictly better off.

Corollary 1 provides a particular case of Proposition 3 when the action set is a compact subset of Euclidean space, the allocation set is a Euclidean space and the principal's expected payoff is strictly decreasing in k and strictly increasing in a .

Corollary 1 Suppose that Assumptions 1 and 2 hold and the optimal solutions to (P2) and (P2) are $(k^*, a^*) \in C^J \setminus C_I^J$ and $(k^*, a^*) \in C_I^J$ respectively. f

Corollary 2 (i) A is a compact subset of R^R and $D = R^q$. $p, q \geq 1$

(ii) (k^*, a^*) is strict EP and

(iii) U is strictly decreasing in k and strictly increasing in a .

then joint-based EP mechanism design strongly dominates both individual-based EP mechanism design and ex post menu design.

Proof. Let $k^l(\cdot) = k(\cdot) \frac{\|f_l/k^*(t)\|}{\|f_l\|} \mathbf{I}_q$, where $\|f_l\|$ is such that $k^l(\cdot) \in K$ for each l and $\|\cdot\|$ denotes the norm of $k(\cdot)$ on the real-vector-valued function space K , and $\mathbf{I}_q$ denotes the unit vector on $\mathbb{R}^q$

Let $a^l(\cdot) = a(\cdot) \frac{\|f_l/a^*(t)\|}{\|f_l\|} \mathbf{I}_R$, where $\|a(\cdot)\|$ denotes the norm of $a(\cdot)$ on A , and $\mathbf{I}_R$ denotes the unit vector on $\mathbb{R}^R$

Obviously, $\{(k^l, a^l)\}_l \in F(T, K, A) \setminus F_{tr}(T, K, A)$.

Moreover, as $l \rightarrow \infty, k^l(\cdot) \rightarrow k(\cdot), a^l(\cdot) \rightarrow a(\cdot)$ for each $\cdot$, and $U(k^l, a^l) \rightarrow U(k, a)$

Hence, Proposition 3 applies here. ■

5. Equivalence of All the Procedures

One may also wonder about some intuitive sufficient conditions under which the joint-based ex post mechanism design, individual-based ex post mechanism design and ex post menu design are all equivalent. The equivalence means that the solutions to all three design problems yield the identical expected payoff for the principal. Again, it suffices to study EPIC mechanisms out of general ex post mechanisms. The delegation principle indicates that this equivalence occurs if some feasible constraint forces the feasible mechanisms to be individual-based, although this case is not observed often in practice. However, even if feasible mechanisms are allowed to be joint-based, there is still some case in which the equivalence occurs.

Proposition 4 Suppose that Assumptions 1 and 2 hold if

(i) $K = \bigcup_{i=1}^n K_i^c$ where K_i^c is a compact subset of K_i for each $i \in N$

(ii) for each i $\varphi_i(f, a, \cdot) = \varphi_i(f_i, a_i, \cdot)$ and

(iii) $\sum_{i=1}^n \varphi_i(f, a, \cdot)(d) = \sum_{i=1}^n \varphi_i(f_i, a_i, \cdot)(d_i)$ where $\varphi_i(f_i, a_i, \cdot)(d_i)$ is the expected payoff that the principal can get merely from agent i ,
then (P2) is equivalent to (P2') and (P3).

Proof. The proof is actually very straightforward. In this case, solving (P2) is equivalent to solving n independent problems simultaneously. The i -th problem is

$$\begin{aligned} & \max_{(\bar{k}_i, \bar{a}_i) \in F(T_i, K_i, A_i)} \varphi_i(\bar{k}_i(\cdot), \bar{a}_i(\cdot), \cdot)(d_i) \\ & \text{s.t. } \varphi_i(\bar{k}_i(\cdot), \bar{a}_i(\cdot), \cdot) \geq \varphi_i(\bar{k}_i(\cdot), a_i, \cdot), \text{ for all } a_i \in A_i \text{ and } \cdot \in T_i, \end{aligned}$$

where $\bar{F}(T_i, K_i, A_i) = M(T_i, K_i) \times M(T_i, A_i)$. In this case, $(\bar{k}, \bar{a})$ will yield the sum of the optimal objective values equal to the optimal objective value of (P2). ■

Hypothesis (i) suggests that feasible contract profiles for all agents are not cross-constrained. Hypothesis (ii) implies that the agents are fully independent, i.e., each agent's expected payoff function only depends on his own contract, action and type. Hypothesis (iii) means that

the principal's expected payoff is additively separable with respect to individual agents. In this situation, the impacts of different agents' asymmetric information on the principal's welfare are independent and separate. The principal can treat the multiple agents as separate and independent individuals in her viewpoint. Even if these agents are heterogeneous, individual-based ex post mechanism design or ex post menu design can still take the same effect as joint-based ex post mechanism design then.

5.4 Centralization vs. Decentralization: Further Discussion

The difference between centralized mechanism design and decentralized menu design becomes more salient in multi-agency than single-agency. Although the individual-based EPIC mechanism is not observed in practice often, it serves as a hub in the examination of centralization versus decentralization in multi-agency contracting. This analysis suggests that the joint-based ex post mechanism design dominates the individual-based ex post mechanism design and the ex post menu design in multi-agency environments. In this respect, centralization on the basis of peer reports (types) will be more desirable than decentralization in contracting, especially when the impacts of different agents' asymmetric information are interrelated.

Nonetheless, my results are based on the existence of (nontrivial) EPIC mechanisms or ex post menus and optimal solutions to all the aforementioned contracting problems. First of all, the existence of (nontrivial) individual-based EPIC mechanisms or ex post menus is the sufficient but not necessary condition for the existence of (nontrivial) joint-based EPIC mechanisms. If the set of (nontrivial) joint-based EPIC mechanisms is empty, the set of (nontrivial) individual-based EPIC mechanisms will definitely be empty too, because the former is actually equivalent to a subset of the latter. The (nontrivial) ex post menu will furthermore not exist by the delegation principle. Then it is meaningless to consider decentralization. In this sense, the principal should always treat centralization in terms of joint-based ex post mechanisms as the starting point to solve her contracting problems.

The analysis of this paper is based on a world free of transaction costs as yet. However, transaction costs normally exist for either using mechanisms or using menus. People must pay attention to the effect of transaction costs in the real world. Since mechanism design entails centralized communication, it may incur higher transaction costs than menu design. In some occasions this extra transaction cost may be remarkably high. For instance, there may be too many agents for the principal to handle, or the principal may not have sufficient technological capacity to process mass data. In that case, the principal may still use menu design instead of joint-based mechanisms in practice.

As long as the transaction costs of using mechanisms is not significantly larger than that of using menus, centralization will win over decentralization. The bottom line is that the increment of transaction cost of using joint-based mechanisms relative to using menus is not beyond the benefit increment of using joint-based mechanisms relative to using menus. In principle,

relative (joint-based) mechanisms are thus of more significance than both absolute (individual-based) mechanisms and menus in contracting games with ex post implementation. Nevertheless, menu design, as a simpler procedure, may still be desirable when relevant transaction costs are remarkable or when the equivalence conditions are satisfied in the analysis of section 5.3.

6 An Application: Financial Regulation

As an aftermath of the recent financial crisis, people tend to pay more attention to economic regulation again. In most contexts, one regulator is facing many regulatees instead of only one regulatee. Nowadays, the impacts of different regulatees' asymmetric information have become more and more interrelated. For instance, think about the bank oligopolists before the financial crisis. As a result, severe exacerbation of asymmetric information has been observed. Giammarino et al. (1993) examine an incentive approach to banking regulation with respect to single agent versus single principal. They simply focus on the decentralized menus of linked options in the regulatory contracting process due to the delegation principle for single-agency. However, decentralization cannot function better than centralization in multi-agency. Let us see a simple application of the main results of this paper in banking regulation with pure adverse selection.

One regulator engages in regulating two duopolistic banks $i = 1, 2$ when transaction cost is negligible. The innate quality of the bank i 's loan portfolio is $q_i \in \{L, H\}$, where L stands for low type and H stands for high type. It is i 's private information. The probability distribution of (q_1, q_2) is given as follows:

$$(L, H) \quad (H, L) \quad \frac{7}{1}; \quad (L, L) \quad (H, H) \quad \frac{1}{1}$$

This suggests that it is most likely that one bank is high type and the other is low type. The regulatory contracting instrument is simply the level of equity financing required, denoted by $e_i \in \{1, 10\}$ for bank $i = 1, 2$

Bank i 's payoff function is $\pi_i(e_i, q_1, q_2)$. The banks' rankings of contracts for each type profile, given by the banks' payoff functions, are as below:

$$\begin{aligned} \pi_1(10, L, H) & \pi_1(1, L, H), \pi_1(10, L, L) & \pi_1(1, L, L), \\ \pi_1(1, H, H) & \pi_1(10, H, H), \pi_1(1, H, L) & \pi_1(10, H, L); \\ \pi_2(10, H, L) & \pi_2(1, H, L), \pi_2(10, L, L) & \pi_2(1, L, L), \\ \pi_2(1, H, H) & \pi_2(10, H, H), \pi_2(1, L, H) & \pi_2(10, L, H) \end{aligned}$$

Assume that all the utilities are positive and the outside option is 0. This means that the individual rationality is automatically met.

The payoff function of the regulator $u(e_1, e_2, q_1, q_2)$ is specified as follows:

$$\begin{aligned} u(10, 10, H, H) &= 15, u(10, 1, H, H) = u(1, 10, H, H) = 13, \\ u(10, 1, L, H) &= u(1, 10, H, L) = 12, u(1, 1, H, H) = 11, \end{aligned}$$

$$\begin{aligned}
& u(10, 10, L, H) \quad u(10, 10, H, L) \quad 10, u(10, 10, L, L) \quad 8, u(10, 1, L, \\
& L) \quad u(1, 10, L, L) \quad , u(10, 1, H, L) \quad u(1, 10, L, H) \quad 4, \\
& u(1, 1, L, H) \quad u(1, 1, H, L) \quad 3, u(1, 1, L, L) \quad 1
\end{aligned}$$

Next, the expected payoff of the regulator given any (e_1, e_2) is

$$U(e_1, e_2) = \frac{7}{1} u(e_1, e_2, L, H) + \frac{1}{1} u(e_1, e_2, L, L) + \frac{7}{1} u(e_1, e_2, H, L) + \frac{1}{1} u(e_1, e_2, H, H)$$

The joint-based direct regulatory contracting mechanism is $E = \{E_i(q_1, q_2)\}_{i=1,2}$. If the banks report their types (q_1, q_2) to the regulator, the level of equity financing required $E_i(q_1, q_2)$ will be specified to each bank i . All the available mechanisms can thus be viewed as all the combination of the ordered tuples (q_1, q_2, e_1, e_2) . In view of the banks' rankings of contracts for each type profile, all individual-based EPIC mechanisms available to the regulator are:

$$\begin{aligned}
\bar{E}^1 & \quad \{\bar{E}_i(H) = 1, \bar{E}_i(L) = 10\}_{i=1,2}, \\
\bar{E}^2 & \quad \{\bar{E}_i(H) = \bar{E}_i(L) = 10\}_{i=1,2}, \\
\bar{E}^3 & \quad \{\bar{E}_i(H) = \bar{E}_i(L) = 1\}_{i=1,2}, \\
\bar{E}^4 & \quad \{\bar{E}_1(H) = \bar{E}_1(L) = 1, \bar{E}_2(H) = \bar{E}_2(L) = 10\}, \\
\bar{E}^5 & \quad \{\bar{E}_1(H) = \bar{E}_1(L) = 10, \bar{E}_2(H) = \bar{E}_2(L) = 1\}, \\
\bar{E}^6 & \quad \{\bar{E}_1(H) = \bar{E}_1(L) = 1, \bar{E}_2(H) = 1, \bar{E}_2(L) = 10\}, \\
\bar{E}^7 & \quad \{\bar{E}_1(H) = \bar{E}_1(L) = 10, \bar{E}_2(H) = 1, \bar{E}_2(L) = 10\}, \\
\bar{E}^8 & \quad \{\bar{E}_1(H) = 1, \bar{E}_1(L) = 10, \bar{E}_2(H) = \bar{E}_2(L) = 10\}, \\
\bar{E}^9 & \quad \{\bar{E}_1(H) = 1, \bar{E}_1(L) = 10, \bar{E}_2(H) = \bar{E}_2(L) = 1\}
\end{aligned}$$

The expected payoff of the regulator given EPIC mechanism E is

$$\begin{aligned}
U(E) &= \frac{7}{1} u(E(L, H), L, H) + \frac{1}{1} u(E(L, L), L, L) \\
&\quad + \frac{7}{1} u(E(H, L), H, L) + \frac{1}{1} u(E(H, H), H, H)
\end{aligned}$$

If one restricts interest merely in individual-based mechanism design, the optimal objective value of $U(E)$ is equal to $\frac{187}{16}$. It will be achieved under a particular joint-based EPIC mechanism $E^1 = \{E_1(L, H \text{ or } L) = 10, E_1(H, H \text{ or } L) = 1, E_2(L \text{ or } H, H) = 1, E_2(L \text{ or } H, L) = 10\}$, which is equivalent to the individual-based EPIC mechanisms $\bar{E}^1$.

Now consider another joint-based mechanism

$$\begin{aligned}
E^0 &= \{E_1(L, L) = 10, E_1(H, H) = 10, E_1(H, L) = 1, E_1(L, H) = 10; \\
&E_2(L, L) = 10, E_2(H, L) = 10, E_2(L, H) = 1, E_2(H, H) = 10\}
\end{aligned}$$

Unlike E^1 , E^0 suggests that the low level of equity requirement is designated for only the

relative high type instead of the nominally high type. Note that E^0 is a kind of perturbation of E^I in some sense. It is easy to verify that E^0 is also EPIC according to the banks' rankings of contracts. E^0 wins over E^I for the regulator, since $U(E^0) = \frac{191}{16} > \frac{187}{16} = U(E^I)$. In such a finite environment, joint-based mechanism design must strongly dominate individual-based mechanism design.

Let us check the menu design next. The set of available contract menu for each bank i is $\{\{1\}, \{10\}, \{1, 10\}\}$. There are nine possible joint menus:

$$\begin{aligned} C^1 &= (\{1, 10\}, \{1, 10\}), C^2 = (\{10\}, \{10\}), C^3 = (\{1\}, \{1\}), \\ C^4 &= (\{1\}, \{10\}), C^5 = (\{10\}, \{1\}), C^6 = (\{1\}, \{1, 10\}), \\ C^7 &= (\{10\}, \{1, 10\}), C^8 = (\{1, 10\}, \{10\}), C^9 = (\{1, 10\}, \{1\}) \end{aligned}$$

They are all ex post (joint) menus according to the banks' rankings of contracts. One can easily compute the banks' EPE response $\langle l(q_1, q_2, C) \rangle$. For each $i = 1, 2, \dots, 9$,

$$\langle l(q_1, q_2, C^h) \rangle = (\bar{E}_1^h(q_1), \bar{E}_2^h(q_2))$$

in fact. Note that C^I yields a (Nash) feasible contract-selection profile for each type profile as follows:

$$\begin{aligned} \langle l(L, H, C^I) \rangle &= (10, 1), \langle l(H, H, C^I) \rangle = (1, 1), \\ \langle l(L, L, C^I) \rangle &= (10, 10), \langle l(H, L, C^I) \rangle = (1, 10) \end{aligned}$$

Apparently, this profile is directly corresponding to $\bar{E}^I$ and E^I , according to $\bar{U}$ and U . Among the nine ex post menus, only C^I induces the optimal objective value for the regulator that is equal to $\frac{187}{16}$. Individual-based ex post mechanism design thus yields a strategically equivalent outcome as ex post menu design, because the delegation principle takes effect here. Moreover, joint-based ex post mechanism design strongly dominates ex post menu design. Therefore, it is more desirable for the financial regulatory contracting to be centralized in terms of joint-based mechanisms than to be decentralized in this multi-agency environment.

7 Concluding remarks

In multi-agency environments, centralized mechanism design can function better than decentralized menu design in contracting games with ex post implementation, as long as the related transaction costs allow it. Centralization has a comparative advantage of using joint-based mechanisms, that is, in specifying contract and action recommendation to each agent, the principal can refer to each agent's report (or true type) relative to all others' reports (or true types), so as to gain more information to deal with the interrelated information asymmetry problem. These findings can be applicable in various multi-agency environments, such as business practices,

organization management, public policy, etc.

There are a number of directions for future research:

1. In case the information structure is fine enough, the agents are still able to play Bayesian Nash games. It is technically demanding to address this situation. But I expect that results similar to those of this paper still hold.
2. The general conditions for existence of ex post implementation or optimal solutions to all contracting problems need to be examined precisely.
3. This analysis scrutinizes only the influences of different contracting procedures on the principal's welfare other than the agents' welfare. The related analysis will require more specific settings.
4. The results of this paper call for further empirical or experimental testing.

8 AppendiE

Proof of Proposition 1

(i) The continuity of $\Phi_i(\cdot, \cdot, \cdot)$ on $K \times A \times T$ follows from Delbaen's Lemma(1974). The proof is similar to Proposition 3.1 in Page (1987).

(ii) The proof for the continuity of $\Psi(\cdot, \cdot, \cdot)$ is similar to (i).

$u(\cdot, f(\cdot), a, \cdot)$ is Borel-measurable on $[! \times ! \times T$, and $u(\cdot, f(\cdot), a, \cdot)$ is bounded for each $\cdot$. By Propositions 7.26 and 7.29 in Bertsekas and Shreve (1978), the mapping

$$\Phi_i(\cdot, \cdot, \cdot) = \int u(w, f(w), a, \cdot) p(dw | a, \cdot)$$

is $B(T)$ -measurable, that is, $\Psi(\cdot, f, a)$ is $B(T)$ -measurable. $\bullet$

Proof of Theorem 1

For any EPE $(p, \cdot)$ of the subgame played by the agents under any joint-based general ex post mechanism $(k, m) \in F(R, K, M)$ in (P1), we have that for each $i \in N$ and each $\cdot \in T$,

$$\begin{aligned} & \Phi_i(k(p(\cdot)), (m(p(\cdot)), \cdot), \cdot) \\ \geq & \Phi_i(k(p_i(\cdot), p_{-i}(\cdot)), \cdot, i(m_i(p_i(\cdot), p_{-i}(\cdot)), i), \\ & \cdot, i(m_{-i}(p_i(\cdot), p_{-i}(\cdot)), \cdot), \cdot), \end{aligned} \quad (9)$$

for all $p_i \in M(T_i, R_i)$ and all $\cdot \in M(M_i \times T_i, A_i)$

Now pick a direct mechanism $(\bar{k}, \bar{a}) \in F(T, K, A)$ such that

$$(\bar{k}(\cdot), \bar{a}(\cdot)) = (k(p(\cdot)), (m(p(\cdot)), \cdot)), \quad (10)$$

for all $\theta \in T$

Observe that such a direct mechanism gives the same expected utility to the principal as the originally given general mechanism, since the probability distribution over decision vectors for any type vector is the same.

Next, we claim that this direct mechanism is EPIC.

(9) implies that for each $i \in N$ and each $\theta \in T$,

$$\begin{aligned} & \mathbb{E}_i(k(p(\theta)), (m(p(\theta)), \theta), \theta) \\ & \geq \mathbb{E}_i(k(p_i(\theta_i), p_{-i}(\theta_{-i})), a_i, \theta_{-i}(m_{-i}(p_i(\theta_i), p_{-i}(\theta_{-i})), \theta_{-i}), \theta), \end{aligned} \quad (11)$$

for all $a_i \in A_i$ and all $\theta_i \in T_i$. This is because for each $a_i \in A_i$ and each $\theta_i \in T_i$ there always exist some $p_i \in M(T_i, R_i)$ and some $\theta_{-i} \in M(M_{-i} \cup T_i, A_{-i})$ such that $p_i(\theta_i) = p_i(\theta_i)$ and $\theta_{-i}(m_{-i}(p_i(\theta_i), p_{-i}(\theta_{-i})), \theta_{-i}) = \theta_{-i}(m_{-i}(p_i(\theta_i), p_{-i}(\theta_{-i})), \theta_{-i}) = a_i$.

(10) and (11) imply that for each $i \in N$ and each $\theta \in T$,

$$a(\theta_i, \theta) \geq \mathbb{E}_i(k(\theta_i, \theta_{-i}), a_i, a_{-i}(\theta_i, \theta_{-i}), \theta),$$

for all $a_i \in A_i$ and all $\theta \in T$

Therefore, this particular direct mechanism (k, a) is EPIC.

Furthermore, the optimal solution to (P1) will clearly bring the same expected utility to the principal as the optimal solution to (P2).

By similar argument, these results also apply to individual-based ex post general mechanisms and individual-based EPIC mechanisms. •

Proof of Lemma 1

$a(\theta) \in T \cup C \cup A$ (f, a) is the EPE under C is closed.

First fix $\theta \in T$. Pick any arbitrary sequence $\{(\theta^a, f(\theta^a), a(\theta^a))\}$ in Grp_C satisfying

$$\begin{aligned} & (f(\theta^a), a(\theta^a)) \in p_C(\theta^a), \\ & \text{and } (\theta^a, f(\theta^a), a(\theta^a)) \rightarrow (\theta, f(\theta), a(\theta)), \text{ as } a \rightarrow \infty \end{aligned}$$

Thus it suffices to show that $(f(\theta), a(\theta)) \in p_C(\theta)$, that is, for each $i \in N$,

$$a_i(\theta_i), a_{-i}(\theta_{-i}), \theta,$$

for all $f_i \in M(T_i, K_i)$ satisfying $(f_i(\theta_i), f_{-i}(\theta_{-i})) \in C$ and all $a_i \in M(T_i, A_i)$

For each $i \in N$,

$$\mathbb{E}_i(f(\theta^a), a(\theta^a), \theta) \geq \mathbb{E}_i(f_i(\theta_i^a), f_{-i}^a(\theta_{-i}^a), a_i(\theta_i^a), a_{-i}(\theta_{-i}^a), \theta),$$

for all $f_i \in M(T_i, K_i)$ satisfying $(f_i(\cdot), f_{-i}(\cdot)) \in C$ and all $a_i \in M(T_i, A_i)$

Then by joint continuity of φ_i ,

$$a_i(\cdot), a_{-i}(\cdot),$$

for all $f_i \in M(T_i, K_i)$ satisfying $(f_i(\cdot), f_{-i}(\cdot)) \in C$ and all $a_i \in M(T_i, A_i)$ •

Proof of Proposition 2

T, C and A are all Borel spaces. $\langle I(\cdot, C) \rangle$ is compact-valued. Its graph is closed in $T \times C \times A$ by Lemma 1. Thus, by Theorem 3 in Himmelberg, Parthasarathy and Van Vleck (1976), $\langle I(\cdot, C) \rangle$ is a Borel-measurable. •

Proof of Lemma 2

T and $K \times A$ are Borel space. By Proposition 2, for each $C \in P_f(K)$ satisfying $\langle I(\cdot, C) \rangle$ is well-defined, $\langle I(\cdot, C) \rangle$ is Borel-measurable and compact-valued. By Proposition 1, ϑ is Borel-measurable and $\vartheta(\cdot, \cdot, \cdot)$ is continuous.

Hence, by Theorem 2 in Himmelberg, Parthasarathy and Van Vleck (1976), there exists some $(\bar{k}, \bar{a}) \in E_{\langle I \rangle}(C)$ such that $\vartheta(\bar{k}(\cdot), \bar{a}(\cdot), \cdot) = \max_{(f(t), a(t)) \in \langle I(t, C) \rangle} \vartheta(f(t), a(t), \cdot)$ for all $\cdot \in T$. Moreover, the function $\vartheta = \max_{(f(t), a(t)) \in \langle I(t, C) \rangle} \vartheta(f(t), a(t), \cdot)$ is also Borel measurable. •

References

- [1] Aliprantis, C. D. and Border, K. C. (2006): *Infinite Dimensional Analysis: A Hitchhiker's Guide*, Springer-Verlag, Berlin-Heidelberg.
- [2] Bergemann, D. and Valimaki, J., (2002): *Information Acquisition and Efficient Mechanism Design*, *Econometrica*, 70, No.3, 1007–1033
- [3] Bergemann, D., Morris, S., (2005): *Robust Mechanism Design*, *Econometrica* 73, 1771–1813.
- [4] Bergemann, D., Morris, S., (2008): *Ex Post Implementation*, *Games and Economic Behavior*, 63, 527–566.
- [5] Bergemann, D., Morris, S., (2009): *Robust Implementation in Direct Mechanisms*, *Review of Economic Studies* 76, 1175–1204
- [6] Bergemann, D., Morris, S., (2011): *Robust Implementation in General Mechanisms*, *Games and Economic Behavior*, Vol. 71, Issue 2, 261–281
- [7] Bertsekas, D. P. and Shreve, S. E. (1978): *Stochastic Optimal Control: the Discrete Time Case*, Academic Press, New York.

- [8] Bikhchandani, S. (2006): "Ex Post Implementation in Environments with Private Goods," *Theoretical Economics* 1, 369-393.
- [9] Carlier, G., (2001): "A General Existence Result for the Principal-Agent Problem with Adverse Selection," *Journal of Mathematical Economics* 35, 129-150.
- [10] Carmona, G. and Fajardo, J. (2009): "Existence of Equilibrium in Common Agency Games with Adverse Selection," *Games and Economic Behavior*, 66, Issue 2, 749-760.
- [11] Chung, K.-S., and Ely, J.C. (2006): "Ex-Post Incentive Compatible Mechanism Design," Discussion Paper, Northwestern University.
- [12] Chung, K.-S., and Ely, J.C. (2007): "Foundations of Dominant Strategy Mechanisms," *The Review of Economic Studies*, 74, 447-476.
- [13] Dasgupta, P. and Maskin, E. (2000): "Efficient Auctions," *Quarterly Journal of Economics*, 115, 341-388.
- [14] Edman, A., Goldstein, I. and Zhu, J., (2011): "Contracting With Synergies," Discussion Paper, University of Pennsylvania.
- [15] Giammarino, R. M., Lewis, T. R. and Sappington, D. E. M. (1993): "An Incentive Approach to Banking Regulation," *The Journal of Finance*, 48, No. 4 pp. 1523-1542.
- [16] Hammond, P. J., (1979): "Straightforward Individual Incentive Compatibility in Large Economies," *Review of Economic Studies*, Vol.46. No.2 263-282.
- [17] Han, S., (2006): "Menu Theorems for Bilateral Contracting," *Journal of Economic Theory* 131, 157-178.
- [18] Himmelberg, C. J., Parthasarathy, T., and VanVleck, F. S. (1976): "Optimal Plans for Dynamic Programming Problems," *Mathematical of Operations Research* 1, 390-394.
- [19] Holmstrom, B. (1982): "Moral Hazard in Team," *Bell Journal of Economics*. 13, No.2, 324-340.
- [20] Holmstrom, B., Milgrom, P., (1991): "Multitask Principal-Agent Analyses: Incentive Contracts, Asset Ownership and Job design," *Journal of Law Economics and Organization*, Vol.7, 24-52.
- [21] Jehiel, P. and Moldovanu, B., (2001): "Efficient Design with Interdependent Valuations," *Econometrica*, 69, No. 5, pp. 1237-1259.
- [22] Jehiel, P., Meyer-Ter-Vehn, M., Moldovanu, B., and Zame, W., (2006): "The Limits of Ex post Implementation," *Econometrica*, 74, No. 3, 585-610

- [23] Kadan,O, Reny, P.J., and Swinkels, J.M., (2011): "Existence of Optimal Mechanisms in Principal-Agent Problems," MF Worsing Paper No. 2011-002.
- [24] Martimort, D. and Stole, L. (2002): "The Revelation and Delegation Principles in Common Agency Games," *Econometrica* 70. Issue 4, 1659-1673.
- [25] Milgrom, P.R. and Roberts, J., (1992), "Economics, Organization and Management," Prentice Hall, Englewood Cliffs, New Jersey.
- [26] Myerson, R.B. (1982): "Optimal Coordination Mechanisms in Generalized Principal-Agent Problems," *Journal of Mathematical Economics* 10, Issue 1, 67-81.
- [27] Myerson, R.B. (1989): "Mechanism Design," in *The New Palgrave: Allocation, Information, and Markets*, edited by J. Eatwell, M. Milgate, and P. Newman, New York: Norton, 191-206.
- [28] Page, Jr. F.H. (1987): "The Existence of Optimal Contracts in the Principal-Agent Model," *Journal of Mathematical Economics* 16, 157-167.
- [29] Page, Jr., F. H. (1992): "Mechanism Design for General Screening Problems with Moral Hazard," *Economic theory* 2, 265-281.
- [30] Page, Jr. F. H. and Monteiro, P. K. (2003): "Three Principles of Competitive Nonlinear Pricing," *Journal of Mathematical Economics* 39, 63-109.
- [31] Perry, M. and Reny, P.J. (2002): "An Efficient Auction," *Econometrica*, 70, No.3, 1199-1212.
- [32] Peters, M. (2001): "Common Agency and the Revelation Principle," *Econometrica*, 69, Issue 5, 1349-1372.
- [33] Ponstein,J., (1966): "Existence of Equilibrium Points in Non-Product Spaces," *S AM Journal on Applied Mathematics*, 14, No.1, 181-190.
- [34] Rosen, J.B., (1965): "Existence and Uniqueness of Equilibrium Points for Concave N-Person Games," *Econometrica*, 33, No.3, 520-534.
- [35] Tulcea, A. I. (1973): "On Pointwise Convergence, Compactness and Equicontinuity in the Lifting Topology I," *Z. Wahrscheinlichkeitstheorie verw. Geb.* 26, 197-205.
- [36] Wilson, R. (1987): "Game-Theoretic Approaches to Trading Processes," in T. Bewley (ed.) *Advances in Economic Theory: Fifth World Congress*, Ch. 2 Cambridge University Press, 33-77.

The Long-run Macroeconomic Impacts of Fuel Subsidies

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March 2013

Abstract

Many developing and emerging market countries have subsidies on fuel products. Using a small open economy model with a non-traded sector I show how these subsidies impact the steady state levels of macroeconomic aggregates such as consumption, labor supply, and aggregate welfare. These subsidies can lead to crowding out of non-oil consumption, inefficient inter-sectoral allocations of labor, and other distortions in macroeconomic variables. Across steady states aggregate welfare is reduced by these subsidies. This result holds for a country with no oil production and for a net exporter of oil. The distortions in relative prices introduced by the subsidy create most of the welfare losses. How the subsidy is financed is of secondary importance. Aggregate welfare is significantly higher if the subsidies are replaced by lump-sum transfers of equal value.

Keywords: oil, fuel-price subsidies, developing countries, fiscal policy
JEL Classifications: Q43, E62, H30, O23

*This is a heavily revised chapter of my thesis that previously circulated under the title “The Long-run Macroeconomic Impacts of Fuel Subsidies in an Oil-importing Developing Country.” For many useful comments and suggestions I give thanks to the editor, two anonymous referees, Nathan Balke, Edward Buffie, Scott Davis, Michael Sposi, and Mine Yucel. Thanks also goes to seminar participants at the MEA 2012 meetings and the USAEE 2012 meetings. Amy Jordan provided excellent research assistance. All mistakes remain my own. The views presented in this paper are mine alone and do not reflect the official views of the Federal Reserve Bank of Dallas or the Federal Reserve System as a whole.

1 Introduction

Subsidies on petroleum products are an important policy issue for many developing and emerging market economies. One reason is the sheer cost these subsidies impose on the governments that provide them. Data from the International Energy Agency (IEA) and the International Monetary Fund (IMF) provide many examples for net oil importing countries where the subsidies are on the order of 1 to 2 percent of GDP, sometimes higher. For net oil exporting countries the subsidies are often larger. Despite their costs, these subsidies are difficult to remove once in place and attempts to remove them, even partially, have often failed. This has been true even with the significant increase in oil prices seen over the last decade.¹

Given their cost and persistence, it seems probable that these subsidies have important macroeconomic implications. This paper asks three inter-related questions in regards to this. First, how do these subsidies affect macroeconomic variables and aggregate welfare in the long-run? Second, what role does the method of financing the subsidy play in those results? Finally, does the distinction between being a net importer or exporter of oil matter?

To answer to these questions I construct a small open economy model with traded and non-traded sectors where households and firms use oil. The government subsidizes oil by selling it below its world price. The subsidy considered in this paper is a permanent (long-run) feature of the economy. As such it distorts the steady state and imposes a permanent financing constraint on the government.²

Two variants of the model are considered. The first is an economy that has no domestic production of oil. This variant is referred to as the net oil importing model. In this model the government finances the subsidy through one of three tax instruments: a non-distorting lump-sum tax, a tax on labor income, or a tax on non-oil consumption. In the second variant, the net oil exporting model, the government has an endowment of oil that is greater than domestic consumption of oil. In this case the government provides the subsidy by simply selling part of its oil endowment below the world price of oil.

¹For more evidence please see Baig et al. (2007) and Coady et al. (2010).

²Some governments do not distort domestic fuel prices in the long-run but do smooth them out in the short-run by temporarily limiting the pass-through of a change in world prices. Chile is one such example. Considering these policies requires looking at short-run dynamics and working with second-order approximations to the model. Given the different nature of such short-run subsidies, this is left for future research.

For the net oil importer case the results show that the subsidy reduces aggregate welfare across steady states.³ For a subsidy that costs 1 percent of GDP the welfare losses are relatively minor but these costs increase substantially for larger subsidies. Surprisingly, the method used to finance the subsidy has relatively little impact on this result. The distortion in relative prices introduced by the subsidy is responsible for the bulk of the welfare losses. This is confirmed by considering the losses that would occur if the government simply removed the subsidy and offset it with lump sum transfers of equal value. Aggregate welfare losses are about 20 times lower under this policy than the one with the subsidy.

In the net oil exporter case the government does not need to rely on an explicit tax to finance the subsidy. Surprisingly, the different financing method available to net exporters does not significantly alter the aggregate welfare results. This is due to the fact that the distortion in relative prices is the main reason that aggregate welfare is lower. That feature of the subsidy is exactly the same whether the country is a net oil exporter or importer.

Underlying the welfare results are the actual changes in macroeconomic variables that occur because of the subsidy. Regardless of how the subsidy is financed, it leads households and firms to over-consume oil products, drives up wages in the economy, and increases production in the traded sector. The subsidy also distorts the relative price of non-tradables to tradeable goods.

The change in other macroeconomic variables, such as non-oil consumption, production in the non-traded sector, and labor supply depends upon the tax instrument used to finance the subsidy. Essentially, households pay for the higher taxes required to finance the subsidy through some combination of lower non-oil consumption and higher hours worked. The exact breakdown depends upon which tax instrument is used because they distort household behavior in different ways. Using labor or consumption taxes to finance the subsidy usually leads to a crowding out of non-oil consumption. This in turn lowers production in the non-traded sector and leads to an inefficient allocation of labor across sectors as labor flows out of the non-traded sector into the traded sector. All of these are important effects of a fuel subsidy typically not discussed by policy makers when considering the pros and cons of the subsidy.

There is a large literature that focuses on oil and the macroeconomy. To my knowledge this is the first paper in that literature that looks at the

³Note these results do not provide any answers about how different groups within the economy are impacted, only how the economy as a whole is. It is quite possible that different groups may have higher or lower welfare depending upon how much of the subsidy they receive and how much of the tax burden they bear, amongst other things.

long-run macroeconomic impacts of fuel subsidies and the fiscal policy issues associated with them.⁴ Several IMF working papers, such as Coady et al. (2006) and Kpodar (2006), have considered the distributional impacts of removing fuel subsidies on household expenditure by using social accounting matrix and input-output models. However, those models generally abstract from the fiscal policy aspect of the subsidy. As a consequence, removing a subsidy is unambiguously “bad” in those models because it means higher prices for all households. My model, which incorporates fiscal policy and general equilibrium effects, suggests things may be more complicated. While removing the subsidy forces households to pay higher fuel prices it also implies lower taxes and reduced deadweight losses in the economy.

A related literature focuses on monetary policy responses to changes in the price of food, another good often subsidized in developing countries. For example, Catao and Chang (2010) explore the role food prices play in determining what price index a central bank should stabilize in a small open economy. Anand and Prasad (2010) consider a similar question in a two sector New Keynesian model where there is a flexible price “food” sector and a “non-food” sector which has sticky prices. Agents who work in the food sector are unable to smooth consumption over time due to a credit constraint. Both of these show that under certain conditions a central bank may want to stabilize headline inflation as opposed to the usual result of stabilizing sticky-price inflation. Neither paper incorporates subsidies into their models.

The rest of the paper proceeds as follows. In the second section I motivate the paper by presenting some data on fuel subsidies. The third section introduces the model economy for the net oil importer case. The results for this case are presented in the fourth section. The fifth section presents results for the net oil exporter case. Section six shows results for sensitivity analysis. Section seven concludes.

2 Empirical Motivation

This section presents some evidence on the prevalence and size of fuel subsidies, and energy subsidies more generally, between the years of 2000 to

⁴Aissa and Rebei (2012) considered optimal monetary policy in a two sector, closed economy New Keynesian model where the government stabilizes the price of one of the goods in the short-run. However, the subsidy in Aissa and Rebei (2012) is a short-run phenomenon only and their analysis focuses on monetary policy, not fiscal policy.

2012.⁵ The main source of data on these subsidies comes from the IEA, the IMF, and several other international agencies. For this reason, I first discuss how these agencies define and measure energy subsidies. I then document some features of the data available from them and conclude by giving more detail on the specifics of energy subsidies in three countries.

2.1 Defining Subsidies

The IEA focuses on subsidies that lower the price consumers pay for oil products, natural gas, coal, or electricity generated with one of those fuels.⁶ These subsidies tend to be the easiest to quantify and in terms of their size appear to be the most important for the time frame being considered. The working definitions of other institutions, such as the IMF, appear to be quite similar in practice so they are not discussed explicitly.

To identify and quantify the size of these subsidies the IEA follows Larsen and Shah (1992) and uses the price-gap approach. In this approach subsidies are measured by calculating the gap between a domestic retail price and a reference price which attempts to measure the true economic cost of the product being subsidized.

Estimates of subsidies calculated using the price-gap approach can reflect both opportunity costs and explicit costs. For a country with no oil production the subsidy is an explicit cost, one typically paid for by the government. For a net oil exporter the subsidy is typically an opportunity cost because in many cases the government simply sells domestically produced oil below its world price. The estimate then simply reflects the foregone revenue from not selling the oil at its economic cost. For a net oil importer with some domestic production the estimate is both an explicit cost and an opportunity cost.

2.2 IEA Data on Fuel Subsidies

Currently the most comprehensive, publicly available data set on energy subsidies is from the IEA. These are annual estimates, in billions of dollars, on the size of consumer subsidies on oil products, natural gas, coal, and electricity generated using fossil fuels in a total of 37 countries. The data begin in 2007 and end in 2011. Here I touch upon some of the more relevant

⁵Data on fuel subsidies in the 1980s and 1990s was sparser. For that reason this section focuses on the time frame mentioned.

⁶For the exact definition please refer to IEA (2010) or IEA (2011).

features of the data for this paper. Those interested in more detail should refer to IEA (2010) or IEA (2011).

For the five years considered, the total value of all energy subsidies across all 37 countries was \$342B, \$555B, \$311B, \$412B and \$523B, respectively. Changes in any given year were to a large extent driven by changes in the price of oil. Subsidies on oil products made up the largest share of the total, on average a little under 50 percent. Out of the 37 countries identified as having a subsidy, 34 had subsidies on oil products, of which 21 were net oil exporters and 13 were net oil importers.⁷

One way to rank which country has large subsidies is by considering the dollar value of the subsidies in place. If one ranks countries by this metric then the biggest subsidizers are generally either net oil importers which have large populations, such as China or India, or important net oil exporters such as Iran or Saudi Arabia. For illustrative purposes the top panel in table 1 ranks the top five net oil importing and exporting countries using the 2011 data as an example.

For the issues considered in this paper a better measure to consider is the size of the subsidies in relation to an economy's GDP. This provides some information on how much of a cost the subsidies impose on the government and the economy in question. The bottom panel of table 1 reconsiders the top five net oil importers and exporters according to this metric using the 2011 data. While absolute size of the subsidy does sometimes predict a large subsidy in relation to the domestic economy, this is not always the case. For example, China's fuel subsidies were huge in dollar terms but in relation to its economy they were fairly small, coming in at a quarter of a percent of GDP. On the other hand, Sri Lanka's subsidies were fairly small in dollar terms, less than a billion dollars, but relatively large in terms of the economy.

Measuring subsidies in relation to a country's GDP highlights an important dichotomy between many net oil exporters and importers. Figure 1 shows this graphically by plotting a histogram with countries categorized by their subsidy-GDP ratio. The data from 2011 is used as an example. For net oil importers every country was between 0 and 2 percent of GDP, except for Egypt. For net oil exporters there was a cluster of countries between 0 and 3 percent of GDP. But there was also an additional cluster of 6 countries which had subsidies between 5 to 9 percent of GDP, as well as an outlier

⁷Countries are defined as net oil exporters or net oil importers using data on annual oil supply and consumption from the Energy Information Administration (EIA) International Energy Statistics.

with subsidies well over 10 percent of GDP. The cluster of 6 countries were all OPEC countries, while the outlier was Iraq.

There are two reasons behind this tendency for subsidies to be larger in net oil exporting countries. First, many net oil exporters tend to subsidize a wider range of products than net oil importers. In many cases all domestically consumed products are subsidized. Holding all else equal, this enlarges the base being subsidized and increases the cost of the subsidy. A second factor is that net oil exporters often have significantly lower retail prices compared to net oil importing countries with subsidies. Taken together these two factors tend to increase the size of the subsidies found in net oil exporters.

For illustrative purposes table 1 and figure 1 used the data from 2011. For those interested in other years or other countries, the data for all 34 countries and 5 years can be found in a table C-4 in the appendix.

2.3 Considering Natural Gas and Electricity Subsidies

While attention often falls on fuel subsidies, both natural gas and electricity subsidies are also important in size. According to the IEA data, subsidies on electricity produced using fossil fuels and subsidies on natural gas averaged close to 27 percent and 23 percent of the total share, respectively, over the 5 years considered.⁸ Natural gas subsidies were identified in 9 countries which were net importers of natural gas and 10 which were net exporters of natural gas. A total of 34 countries were identified as having electricity subsidies.

From an individual country's perspective, natural gas subsidies could often be modeled in a similar fashion to fuel subsidies. For example, consider the case of a net importer of natural gas with a subsidy. The government purchases natural gas at a price determined in a market (often linked to oil prices), sells it below cost, and must finance this through some form of taxation. If the country is a net exporter of natural gas then the government can finance the subsidy by selling domestically produced natural gas below its market price, in which case the subsidy is an opportunity cost.

In many cases electricity subsidies can also be treated in a similar manner. While electricity itself is often not traded between countries, it is in many cases generated using imported inputs such as oil products and natural gas. In this case an electricity subsidy is basically an indirect subsidy on the consumption of the imported fuel. For a net importer of the input, the end result is essentially the same for the government in question, and

⁸Consumer subsidies on coal were negligible in size and are henceforth ignored.

likewise for the government of a net exporting country.

In what follows, I continue to refer generically to fuel subsidies. However, in many cases this could be interchanged with natural gas subsidies or electricity subsidies and the general results should carry over for those cases. Two examples are provided in section 2.5 which highlight the similarities between the different types of subsidies.

2.4 Additional Sources of Data

Additional sources allow one to expand the IEA data by including more countries and considering years prior to 2007. For some countries, additional data on the costs of their subsidies is also available. Using the larger set of sources, I identified another 21 countries that had a fuel subsidy at some point in time between 2000 to 2012.⁹ For subsidies on electricity produced using fossil fuels an additional 15 countries were identified. No other countries were found to have natural gas subsidies besides those listed in the IEA data.

Unfortunately, it is not possible to expand the quantitative data from the IEA for all of the additional countries or years. There are some countries with fairly detailed data on the costs of the subsidies, but in many cases the source documents mention the existence of a subsidy but provide no data on its cost. Consequently, there are too many gaps to construct a comprehensive estimate of the costs across countries and across time. However, a table in the appendix shows the subsidy costs, as a share of GDP, for the countries and years in which that data was available.

2.5 Country Experiences

Indonesia, Ukraine, and Lebanon provide useful examples of how different types of energy subsidies operate and what their costs are to the governments that choose to put them in place. Indonesia subsidizes a wide range of oil products; Ukraine subsidizes household consumption of natural gas; Lebanon subsidizes electricity, almost all of which is produced using imported oil products.

⁹A list of these countries and the sources used to identify them can be found in the appendix.

Fuel Subsidies in Indonesia¹⁰

Indonesia has had experience with subsidies on oil products as both a net exporter and net importer of oil.¹¹ Indonesia's national oil company, Pertamina, is heavily involved in the production, importation, and distribution of oil products in the country. Domestic retail prices are set by the government on an ad hoc basis and subsidies have been in place since the late 1960s. The subsidies take the form of explicit underpricing of the products compared to their actual cost. Pertamina is compensated for this and the costs of the subsidy are reflected in the Indonesian budget.

Most products are currently subsidized, except for some premium grades of gasoline. Using IMF data it is possible to get estimates on the costs of these subsidies, as a percent of GDP, from 2000 to 2012. These are listed in the second column of table 2. The subsidies have been more than 1 percent of GDP each year, and in many cases well above that.

Electricity Subsidies in Lebanon¹²

Lebanon provides a very good example of a country with subsidies on electricity produced using an imported fossil fuel. On average, about 94 percent of its electricity was generated using imported oil products between 2000 - 2010.

The state-run electric utility company, Electricite du Liban, has received direct transfers from the government every year since 1984. These transfers are often used to cover the gap between the cost of the imported fuel and the revenues the company generates from underpriced electricity. Electricity tariffs have been frozen since 1996 and are priced for \$21 a barrel oil, according to an estimate in IMF (2012).

Data from the World Bank and the IMF allow one to calculate the cost of the subsidy, as a percent of GDP, from 1984 up to 2012. The third column of table 2 shows how the costs have varied from 2000 until 2012. The subsidy cost about 1 percent of GDP in the early part of the decade, but since 2005 has been roughly 3 percent of GDP or higher. The 2011 and 2012 estimates from the IMF come in at 4.5 percent of GDP. Projections up until 2016 currently put the cost over 4 percent of GDP each year.

Natural Gas Subsidies in Ukraine¹³

Ukraine is a net importer of natural gas with some domestic production. A state-owned company, Naftogaz, is heavily involved in the production,

¹⁰Sources: EIA (2011), EIA (2012), Clements et al. (2003), IISD (2012a), IISD (2012b) OECD (2010).

¹¹Indonesia became a net importer of oil and oil products in 2004.

¹²Sources: EIA (2012), IMF (2012), World Bank (2008).

¹³Sources: EIA (2012), IEA (2012), Mitra and Atoyan (2012), Petri et al. (2002).

importation, and distribution of natural gas in the country. The gas is consumed by both industry and households and is also used to generate electricity. Household consumption of natural gas is heavily subsidized, with tariffs well below import cost. Industry usage is not subsidized and firms pay a price that reflects import costs.

Measuring the cost of natural gas subsidies in Ukraine is difficult as Naftogaz's activities are quasi-fiscal in nature and the cost of the subsidies has not always been fully reflected in the government's budget. However, the IEA dataset provides a dollar amount for these subsidies for the years from 2007 to 2011. For these five years the subsidies totaled \$4.8B, \$8.3B, \$5.3B, \$5.2B, and \$6.7B, respectively. As a share of GDP this translates to 3.4 percent, 4.6 percent, 4.5 percent, 3.8 percent, and 4 percent.

3 The Model for the Net Oil Importer

I consider a small open economy that produces a composite traded good and a non-traded good. Both goods are produced using labor and oil and one sector may be more or less oil-intensive than the other. The traded good is the numeraire and for convenience its price is fixed at unity. The traded good is either consumed by households or used to purchase oil from the rest of the world. The economy is small in that it has no effect on the world price of the traded good or the world price of oil.

The notation used in the exposition is as follows. The time derivative of the variable X is $\dot{X}$, $\bar{X}$ is the steady state value of X , and $\hat{X}$ is the log-differential of X , i.e. $\hat{X} = dX/X$.

3.1 Households

Household activity is controlled by an infinitely-lived representative agent who derives disutility from working and utility from the consumption of traded and non-traded goods, as well as fuel products.

Total labor supply is denoted as $L = L^T + L^n$ where L^T and L^n are labor supplied to the traded and non-traded sector, respectively. Consumption of the traded, non-traded, and oil goods are denoted as C^T , C^n , and O^h , respectively. The agent has access to a real domestic bond, denoted as b . The representative agent assumption implies this will be in net zero supply in equilibrium. Households do not have access to international capital markets.¹⁴

¹⁴Currency substitution is a feature of many of the countries that have fuel subsidies.

Preferences are given by

$$U = \int_0^\infty \left[\frac{C(C^T, C^n, O^h)^{1-\frac{1}{\tau}}}{1-\frac{1}{\tau}} - \kappa V(L) \right] e^{-\rho s} ds, \quad (1)$$

where

$$C(C^T, C^n, O^h) = \left(C^T^{\frac{\sigma_c-1}{\sigma_c}} + a_1 C^n^{\frac{\sigma_c-1}{\sigma_c}} + a_2 O^h^{\frac{\sigma_c-1}{\sigma_c}} \right)^{\left(\frac{\sigma_c}{\sigma_c-1} \right)},$$

$$V(L) = \frac{L^{1+\frac{1}{\mu}}}{1+\frac{1}{\mu}}.$$

The parameter τ is the elasticity of intertemporal substitution; μ is the Frisch elasticity of labor supply; σ_c is the elasticity of substitution between the consumption goods, ρ is the time-preference rate; a_1 , a_2 , and κ are constants.

The agent maximizes equation(1) subject to the flow constraint

$$\dot{b} = (1-\tau^l) (W^n L^n + W^T L^T) + Tr + rb - (1+\tau^c) (C^T + P^n C^n) - P^s O^h - T. \quad (2)$$

Income from labor is given by $W^T L^T + W^n L^n$ where W^T and W^n are the wages in the traded and non-traded sectors. This income is taxed at a rate of τ^l . Interest income on savings is given by rb and will be zero in equilibrium. Lump-sum transfers from the government are given by Tr . Expenditure on non-oil consumption is given by $C^T + P^n C^n$ where P^n is the relative price of the non-traded good to the traded good. This consumption is taxed at a rate of τ^c . Lump-sum taxes are given by T . Expenditure on oil consumption is given by $P^s O^h$, where P^s is the subsidized price of fuel products. Denoting P^o as the world price of oil, the assumption is that $P^s \leq P^o$.

The first order conditions for the agent's problem can be written as¹⁵

$$\frac{U_n}{U_T} = P^n, \quad (3)$$

$$\frac{U_o}{U_T} = \frac{P^s}{1+\tau^c}, \quad (4)$$

$$\frac{\kappa V_l}{U_T} = \frac{1-\tau^l}{1+\tau^c} W^T, \quad (5)$$

To simplify the exposition I abstract from this possibility in the model. For some results with a model that incorporates this feature please see the previous version of this paper.

¹⁵Please see the appendix for the exact forms of the first order conditions

$$W^n = W^T, \quad (6)$$

$$\frac{\dot{\lambda}}{\lambda} = \rho - r, \quad (7)$$

where λ is the multiplier on the flow constraint and U_n , U_T , and U_o are the derivatives of the utility function with respect to the non-traded, the traded, and the oil consumption good.

Equation (3) sets the marginal rate of substitution between traded and non-traded consumption goods equal to their relative price while equation (4) does the same for the oil consumption good. Equation (5) equates the marginal dis-utility of working an additional hour equal to the marginal benefit of doing so. Equation (6) states that wages are equivalent across the traded and non-traded sectors. This is because of the assumption that labor is mobile across sectors.

The subsidy directly distorts the first order conditions through the relative price term, P^s , in equation (4). There are additional distortions if consumption taxes or labor taxes are used to finance the subsidy, or if the subsidy impacts wages or the relative price of the non-traded good. All of these distortions will impact household consumption and labor supply decisions.

3.2 Production

Production in the two sectors is done by representative firms operating under perfect competition. The firms have a CES technology of the form

$$Q^i(L^i, O^i) = \left[\left(A^i L^i \right)^{\frac{\sigma_i - 1}{\sigma_i}} + b^i \left(O^i \right)^{\frac{\sigma_i - 1}{\sigma_i}} \right]^{\frac{\sigma_i}{\sigma_i - 1}}, \quad (8)$$

where $i = T, n$ for the traded and non-traded sectors, A^i and b^i are constants, O^i is oil demanded by sector i , and σ_i is the elasticity of substitution between value-added (here labor) and oil.

The first order conditions for the firms are given by

$$Q_l^T = W^T, \quad (9)$$

$$Q_o^T = P^s, \quad (10)$$

$$Q_l^n = \frac{W^n}{P^n}, \quad (11)$$

$$Q_o^n = \frac{P^s}{P^n}, \quad (12)$$

where Q_l^i and Q_o^i are the derivatives of the production functions with respect to labor and oil. The first order conditions equate the marginal products of each input with its respective marginal cost. The relative price term appears in the first order conditions for the non-traded sector due to the choice of the numeraire.

Cost functions

The functional form for the production function implies that unit costs for each firm, denominated in terms of the traded good, are

$$\Phi^i(W^i, P^s) = \left[\left(\frac{W^i}{A^i} \right)^{1-\sigma_i} + b^{i\sigma_i} (P^s)^{1-\sigma_i} \right]^{\frac{1}{1-\sigma_i}} \quad (13)$$

for $i = T, n$. Furthermore, the relative price of the non-traded good is given by

$$P^n = \frac{\Phi^n}{\Phi^T}. \quad (14)$$

One can derive two additional and very useful conditions using these cost functions and the equation for P^n .

Facing a constant world price for its output and under the assumptions made regarding production, the real unit cost in the traded sector, Φ^T , is equal to 1. Using this condition, one can immediately show that wages in the traded sector will increase if P^s is lowered in the long-run. More specifically, for small changes in P^s the change in the wage is given by

$$\hat{W}^T = -\frac{\alpha_o^T}{\alpha_l^T} \hat{P}^s, \quad (15)$$

where α_o^T and α_l^T are the cost shares of oil and labor in the traded sector.

Intuitively, lower energy costs would allow firms in the traded sector to sell their output below the world price of the traded good. This would increase demand for their good, and to meet this demand the firm would need to use more labor. The only way to attract this labor is for wages to increase. In the new long-run equilibrium the firm increases its production, and its demand for labor, until the point where its cost of producing an additional unit of output would once again equal the world price of the traded good. Equation (15) provides the exact change in wages required to ensure that this condition holds. The more oil-intensive the traded sector is the greater the increase in wages will be.

The household's first order condition in equation (6) implies that the change in W^T spills over into the non-traded sector. The increase in W^T ,

therefore, acts as a negative cost shock for the non-traded sector. Holding all else equal, this would drive up P^n . But, the non-traded sector also faces lower energy costs because it benefits from the subsidy. In the end which one of these forces wins out depends upon how oil-intensive the non-traded sector is. For a small change in P^s the change in P^n is given by

$$\hat{P}^n = \left(\alpha_o^n - \frac{\alpha_l^n \alpha_o^T}{\alpha_l^T} \right) \hat{P}^s, \quad (16)$$

where α_o^n and α_l^n are the cost shares of oil and labor in the non-traded sector. If the non-traded sector is oil-intensive enough then reductions in P^s reduce costs so much that P^n declines. Otherwise the increase in wages drives up costs and P^n increases.

3.3 The Government

The government earns revenue from lump-sum taxes, taxing labor income and taxing the consumption of non-oil consumption goods.¹⁶ On the expenditure side, the government provides a subsidy on fuel products and lump-sum transfers to households. The government purchases oil at the world price of P^o and then sells it at the subsidized price P^s , with $P^s \leq P^o$. While simple in nature, this assumption regarding the subsidy captures the important fact that domestic prices are lower than world prices and that the subsidy must be financed by the government somehow.

In the steady state the government budget constraint reads

$$\bar{T} + \tau^l (\bar{W}^T \bar{L}^T + \bar{W}^n \bar{L}^n) + \tau^c (\bar{C}^T + \bar{P}^n \bar{C}^n) = \bar{T}r + (\bar{P}^o - \bar{P}^s) (\bar{O}^h + \bar{O}^T + \bar{O}^n). \quad (17)$$

3.4 Market Clearing and the Current Account

Market clearing in the non-traded sector implies

$$C^n = Q^n. \quad (18)$$

In the bond market the equilibrium condition is

$$b = 0, \quad (19)$$

both in and out of the steady state.

In the long-run trade balances so

$$\bar{Q}^T = \bar{C}^T + \bar{P}^o (\bar{O}^h + \bar{O}^T + \bar{O}^n). \quad (20)$$

¹⁶Consumption and income taxes were chosen on the basis of IMF country reports which showed that these two forms of taxation tend to be important sources of revenue for many developing countries. This is particularly true of taxes on goods and services.

3.5 Calibration

The model is calibrated to an initial steady state where there are no subsidies. Parameters and variables are calibrated to match features of a typical developing country that has had experience with fuel subsidies, such as Bangladesh, Sri Lanka, or Indonesia. Units are chosen so that real GDP is equal to 1, as are P^o and P^n . Lump-sum transfers, Tr , are set to 0. Total hours worked is set to $1/3$. Table 3 lists the calibration of the model's other parameters and variables, along with short comments on the sources used in the calibration. Greater detail regarding the sources can be found in the appendix. The calibration of several elasticities and the amount of oil used by firms is discussed in more detail below.

- Elasticities of substitution ($\sigma_c, \sigma_n, \sigma_T$) - These parameters pin down the price-elasticity of demand for oil products. A survey done in Graham and Glaister (2002) shows that long-run elasticities for many countries tend to cluster around -.6 to -1 and that these are larger than short-run elasticities. The baseline calibration is in line with these findings. An alternative calibration of .25, more consistent with short-run price elasticities, is also considered.
- Frisch elasticity of labor supply (μ) - This parameter controls how responsive labor supply is to changes in wages, with larger values implying a greater responsiveness. Even for developed countries there is significant uncertainty surrounding this parameter. Reichling and Whalen (2012) provides a useful summary of the findings. Micro estimates tend to be smaller than macro estimates, often between 0 and 1, as an individual's hours worked tends to be unresponsive to changes in wages. Macro estimates are often between 2 and 4, reflecting the fact that in aggregate data this elasticity is capturing changes in both the intensive and extensive margins on how much people work. The baseline calibration sets μ equal to 1, but an alternative calibration of 3 is also considered.
- Firm demand for oil - Input-output tables for 11 developing countries are available from the OECD STAN database. These tables provide data on spending by firms on "Coke, refined petroleum products, and nuclear fuel." This provides the best estimate on total firm spending on oil products. In theory, one would like to have an estimate for spending only on "refined petroleum products" but unfortunately that data is not available. The average value was close to 5 percent of GDP for all firms. About 40 percent of that was due to firms in the traded sector

(defined as agriculture, mining, and manufacturing).

4 Results

Numerical solutions are calculated for how the model's variables change conditional on the method of financing the subsidy and the size of the subsidy, in relation to initial GDP. I consider subsidy costs that range from 1 to 4 percent of GDP. For each level of the subsidy, one of the tax instruments adjusts to clear the government budget constraint. Labor taxes, consumption taxes, or lump-sum taxes are each considered in turn. With three fiscal instruments this results in a total of 12 cases considered.

The analysis of the results focuses on how the model's variables change across steady states and on how aggregate welfare is impacted by the changes in those variables. Changes in the variables are calculated as percent changes across steady states, where the initial steady state (with no subsidy) is the point of reference.

If X_o and X_1 are the steady state values of variable X in the original and the new steady state, respectively, then the change in aggregate welfare across steady states is given by

$$\frac{1}{\rho} \left\{ \left[\frac{C(C_o^T, C_o^n, O_o^h)^{1-\frac{1}{\tau}}}{1-\frac{1}{\tau}} - \kappa V(L_o) \right] - \left[\frac{C(C_1^T, C_1^n, O_1^h)^{1-\frac{1}{\tau}}}{1-\frac{1}{\tau}} - \kappa V(L_1) \right] \right\}.$$

However, looking at the change in aggregate welfare across steady states is not very informative since utility is ordinal. To make the comparisons more concrete, I solve for how much aggregate consumption in the initial steady state would need to be increased or decreased, in percentage points, to make welfare equal across the two steady states. Mathematically I solve for ω in the following equation,

$$\left\{ \frac{[\omega C(C_o^T, C_o^n, O_o^h)]^{1-\frac{1}{\tau}}}{1-\frac{1}{\tau}} - \kappa V(L_o) \right\} - \left\{ \frac{[C(C_1^T, C_1^n, O_1^h)]^{1-\frac{1}{\tau}}}{1-\frac{1}{\tau}} - \kappa V(L_1) \right\} = 0. \quad (21)$$

In general ω will be non-zero as welfare will be higher or lower across steady states. The welfare losses are calculated as

$$W_l = 100 * (1 - \omega). \quad (22)$$

One way to interpret W_l is as follows. If W_l is positive then aggregate welfare is higher in the initial steady state. Intuitively, aggregate consumption would need to be lowered by W_l percent to match the lower utility in the new steady state. Alternatively, W_l tells one how much aggregate consumption would need to be increased to make the agent as well off in the new steady state as in the initial steady state.

When households and firms receive the subsidy, the solutions combine the effects brought about by changes in the relative price that households face for O^h and changes in the relative price that firms face for O^T and O^n . It is instructive to consider each of these channels separately. To do so I first solve for a case where the household pays the subsidized price but the firm pays the world price of oil. Second, the case where the firm receives the subsidy but households do not is considered. Finally, I present the results under the situation where both households and firms receive the subsidy.

4.1 Households Receive the Subsidy

In many countries, households are often specifically targeted as the beneficiaries of the subsidy. This is particularly true for certain oil products as well as electricity and natural gas, which are harder to divert from their intended recipients. For this reason, it is interesting to first consider the results under the assumption that only households receive the subsidy. Doing so only requires assuming firms pay the prevailing price for oil products, P^o , and the government only needs to raise revenue for under-pricing fuel products to households. All other aspects of the model remain the same.

The top panel in table 4 presents the results for aggregate welfare. Each row is for a different subsidy cost while columns two through four record the results for the different tax instruments. The numbers in each entry tell us how much aggregate consumption in the initial steady state would need to be decreased to get welfare equal across steady states. For example, if the subsidy costs 4 percent of GDP and is financed by a consumption tax W_l is equal to 1.4. This means that aggregate consumption in the initial steady state would need to be 1.4 percent smaller to match the (lower) aggregate welfare of the new steady state.

Regarding the subsidy's impact on aggregate welfare, the numbers in this table are all positive, so aggregate welfare is always lower with the subsidies. In consumption-equivalent terms the welfare costs are fairly small for subsidies on the order of one percent of GDP, coming in at little over a tenth of a percent of aggregate consumption. However, the losses steadily increase as the subsidy reaches higher levels and W_l is well over 1 percent

by the time the subsidy reaches 4 percent of GDP.

Interestingly, the method of financing the subsidy is of secondary importance to the subsidy itself in terms of the welfare losses. There are differences between the different tax instruments but they are fairly small. In general, lump-sum taxes or labor taxes are slightly preferred to financing the subsidy through a consumption tax.

Underlying the welfare results are the actual changes in the variables across steady states. The top panel in table 5 shows these results. Each column is for a different tax instrument. For brevity's sake the results are shown only for the case where the subsidy costs 1 percent of GDP. Larger subsidies would increase the size of the changes that occur, so one should view the quantitative results in the table as a sort of lower bound. Note also that if one wanted to consider the implications of going from the steady state with the subsidy to the steady state without the subsidy, one needs only flip the signs on the results found in the table.

For the baseline calibration a subsidy of 1 percent of GDP leads to a reduction in the subsidized price of fuel by about 26 percent.¹⁷ Regardless of how the subsidy is financed, lower fuel prices induce households to consume roughly 25 percent more fuel.

How the other variables change across steady states depends upon the method of taxation used. Essentially, households pay for the increased taxes by consuming less non-oil goods, working more, or both. The exact breakdown depends upon which tax instrument is used because different tax instruments distort household behavior in different ways.

Lump-sum taxes reduce disposable income but do not otherwise change the effective prices that households face and for this reason they make a useful baseline case. When the subsidy is financed by lump-sum taxes, households both consume less and work more. Consumption of non-oil goods declines by half a percentage point while hours worked increases by about half a percent.

By comparison, labor taxes reduce the incentive to work so hours worked increase less. However, as a consequence, households cut back consumption of non-oil goods by a greater amount. Consumption taxes not only reduce the incentive to work but also further distort the relative price of the consumption goods. Compared with lump-sum taxes, hours worked increase by much less, consumption of oil products rises more, and consumption of other

¹⁷The change in P^s is a mainly a function of the specified subsidy cost and how big the base being subsidized is. If a great deal of oil is consumed by the economy, then small changes in P^s are sufficient to reach a specified level of subsidy cost and vice-versa.

goods falls by a greater amount. The fact that the consumption tax introduces two distortions appears to be the reason it generates slightly higher welfare losses than the other two forms of taxation.

Although firms do not benefit from the subsidy, production variables are impacted indirectly. Reduced demand for the non-traded good causes production in that sector to decline, which lowers that sector's demand for labor and oil. In the long-run this labor flows into the traded sector. Despite labor re-allocating across sectors, wages remain unchanged as the traded-sector faces a constant real unit cost and a constant price P^o for oil inputs. What does happen is that the traded sector expands production by increasing its use of oil and labor proportionally.¹⁸

4.2 Firms Receive the Subsidy

Now suppose that firms pay the reduced price of P^s for oil while households continue to pay the world price P^o . The middle panel in table 4 shows the welfare results for this case. The format of the table is exactly the same as before. Each entry in the table is positive, showing that aggregate welfare is reduced by the subsidy. The losses are fairly small for subsidies on the order of one to two percent of GDP but start growing as the subsidy reaches higher levels. The choice of the tax instrument is again essentially irrelevant for the results.

The middle panel of table 5 shows how the variables change across steady states. With lower energy prices, production expands in the traded sector which brings about higher wages and greater labor usage in that sector. The intuition behind these results is exactly as explained in section 3.2.

For the non-traded sector, things are a little more complicated. As explained in section 3.2 the higher wages drive up costs in this sector but the subsidy lowers costs. For the baseline calibration, the non-traded sector is more oil-intensive than the traded sector. The end result is that lower energy costs drive down unit costs enough to override the increases caused by higher wages in the economy. As a result, there is a small decline in the relative price of the non-traded good to the traded good. In response to this reduction in P^n households are induced to consume more of the non-traded good leading output to expand in that sector as well.

While households do not directly receive the subsidy they do have to pay higher taxes. Which tax instrument is used to pay for the subsidy has some

¹⁸Demand for the two inputs increases proportionally because the firm faces a constant ratio of input prices. This can be seen by combining the first order conditions for labor and oil in the traded sector, equations (9) and (10).

implications for how consumption and hours worked respond. The decrease in P^n leads to increased consumption of the non-traded good regardless of the tax instrument used. However, when labor taxes or consumption taxes are used, consumption of the traded good gets crowded out. The consumption tax induces households to substitute away from the taxed goods into oil. Consumption and labor taxes also reduce the overall increase in hours worked by households compared to the lump-sum case.

Comparing the top and middle panels of table 4 shows that the welfare losses are lower when firms receive the subsidy than when households receive it. This is driven to some extent by the fact that the subsidies are being measured by their size in relation to GDP. The baseline calibration has firms using more oil than households, so a smaller drop in P^s is needed to reach a given subsidy cost. This can be confirmed by comparing how P^s changes in the two cases using table 5. An alternate method would be to calculate the welfare costs as a function of P^s instead of the subsidy cost. Figure 2 plots the welfare losses as a function of P^s for the three cases considered. The dashed line is for the case where households receive the subsidy, the solid line for the case where firms receive the subsidy, and the dashed-dotted line for the case where both receive the subsidy. This figure shows that for a given reduction in P^s , the losses are larger in cases with a bigger base being subsidized.

4.3 Households and Firms Receive the Subsidy

The bottom panel of table 4 records the welfare losses under the scenario where both households and firms receive the subsidy. In general the results are similar to the previous two cases. Comparing the three panels shows that the losses are smaller in this case than in the other two cases. This is explained by the fact that for a given subsidy cost the reduction in P^s is lowest in this case as the base being subsidized is the largest.

All of the movements in the variables are now combinations of the results for the two previous cases. Households face a lower relative price for the oil good and a slightly lower relative price for the non-traded good (compared to the traded good). Households increase their consumption of fuel products and reduce their consumption of traded goods. With P^n lower, consumption of the traded good falls relatively more than the non-traded good. Both labor and consumption taxes reduce hours worked and consumption of non-oil products relative to the lump-sum case. Likewise, consumption taxes drive up consumption of oil products at the expense of non-oil consumption compared to the lump-sum and labor tax case.

4.4 Transfers vs. Subsidies

Aggregate welfare is impacted by both the subsidy and the taxes necessary to finance the subsidy. One result shown in table 4 is that only minor differences are found for the change in aggregate welfare regardless of whether lump-sum taxes, labor taxes, or consumption taxes are used to finance the subsidy. One hypothesis for why this is the case is that the distortions in relative prices introduced by the subsidy are the main reasons that aggregate welfare is lower and not because of changes in T , τ^l , or τ^c .

To consider whether this might be the case I ask what the implications would be on aggregate welfare if instead of increasing subsidies by a certain percent of GDP the government increased lump-sum transfers by the same amount. This would require changing τ^l or τ^c by a similar amount to pay for the increased government spending on Tr but would keep P^s equal to the world price of oil.

The specific experiment considers transfers on the order of 1 to 4 percent of initial GDP. These are financed with either labor or consumption taxes. Table 6 compares the aggregate welfare losses between increased transfers and the subsidy. For brevity's sake only the results from the case where households and firms receive the subsidy are presented. The results are stark: the losses under the subsidy are about 20 to 25 times greater than under a system of transfers. While the increased taxation required on an aggregate level is similar in both cases the distorted relative price significantly increases welfare losses under the subsidy. Similar results are seen in the cases where only households or firms benefit from the subsidy.

This result might be of more than just theoretical importance. In general, it has been quite difficult for countries to remove fuel subsidies once they are in place. To increase the likelihood of a successful reform the IMF has often suggested their removal be offset with increased transfers. Generally it is argued that these transfers can be better targeted to the needy and are often less costly than the fuel subsidies they replace. The results in this paper offer an additional reason for considering this option: giving people money and allowing them to spend it where they want is significantly more efficient than inducing them to consume more fuel products by artificially lowering fuel prices.

5 The Case of a Net Exporter

A large number of net oil exporters also subsidize fuel products. An important reason for considering the case of net oil exporters separately is that

they have an additional financing method available to them. The governments of these countries can simply sell domestically produced oil below its world price. This creates an opportunity cost for the government but negates the need to raise taxes to finance the subsidy.

To address whether this significantly alters the previous results about aggregate welfare this section modifies the baseline model by assuming the economy has an endowment of oil large enough to make it a net exporter of oil. The punchline of the work is that the results are surprisingly similar to the case of the net oil importer. The reason for this is that whether one is considering a net oil exporter or importer the subsidy distorts the relative price of fuel products and it is that distortion which drives most of the welfare results, not the method of financing the subsidy.

5.1 Households and Firms

No changes are made to the assumptions regarding household and firm behavior. Preferences and the budget constraint of the household are assumed to be equivalent for the net oil importer and exporter. Technology in the traded and non-traded sectors is also the same.

5.2 The Government

The major change to the model comes on the government side. In many net exporting countries the government is heavily involved in the production of oil. In line with this, I assume the government now has access to an endowment of oil, Q^o . I assume there is zero cost associated with the supply of oil.¹⁹ The government earns revenue from selling oil domestically to households and firms at a subsidized price of P^s and exports the remaining supply at the world price, P^o . All revenue from the sales of oil, whether from exports or domestic sales, is funneled to the economy through a lump-sum transfer. For simplicity I assume the government does not levy taxes on either labor income or consumption nor does it spend the money on anything besides the transfer. Under those assumptions the steady state government budget constraint reads

$$\bar{P}^o (\bar{Q}^o - \bar{O}^h - \bar{O}^T - \bar{O}^n) + \bar{P}^s (\bar{O}^h + \bar{O}^T + \bar{O}^n) = \bar{T}r. \quad (23)$$

¹⁹This assumption potentially inflates the revenue the government receives from the sale of oil. However, I also considered a case where there was a fixed cost per unit of oil. The results were not significantly different.

There is a subtle point in the budget constraint worthwhile pointing out. If the net oil exporter has no subsidies in place, then government revenue is maximized at $P^o Q^o$. With a subsidy in place, government revenue is reduced below this amount. But this automatically means lower transfers to the economy. In a very real sense, the subsidy is actually being financed by a reduction in lump-sum transfers.

5.3 Market Clearing and the Current Account

Market clearing in the non-traded sector and the bond market are equivalent for the net oil importer and exporter.

The current account equation, however, is different. In the steady state the current account for the net oil exporter reads

$$\bar{Q}^T + \bar{P}^o \bar{Q}^o = \bar{C}^T + \bar{P}^o (\bar{O}^h + \bar{O}^T + \bar{O}^n). \quad (24)$$

Essentially, the net oil exporter has extra oil and uses that to trade for C^T whereas the net oil importer produces extra Q^T and trades that for oil.

5.4 Calibration

The model is calibrated to an initial steady state where there are no subsidies. Where possible the calibration found in table 3 is used for the model of the net oil exporter as this facilitates comparison between this case and the net oil importer case. The deep parameters, given by ρ , τ , σ_c , σ_T , σ_n , and μ are the same across models. Consumption expenditure shares and firm use of oil as a percent of GDP are also calibrated to the same starting values.

I calibrate Q^o as a proportion of total domestic consumption of oil. To guide the calibration I used EIA data on “Total Oil Supply” and “Total Oil Consumption” for Ecuador, a net exporter with large fuel subsidies. For that country, total oil supply was on average about 2.4 times the size of total oil consumption between 2007 and 2011 (the years for which data is available on subsidies in the country).

5.5 Results

Numerical solutions are calculated for how aggregate welfare varies across steady states, depending upon the size of the subsidy as a fraction of initial GDP. This is the same procedure that was done for the net oil importer.

Table 7 presents the welfare results.²⁰ For brevity's sake only the results for the case where households and firms receive the subsidy are presented. The second column shows the welfare costs for varying levels of the subsidy compared to the baseline case where there are no subsidies. While the method of finance is quite different from what occurs in the net importer model, the results are surprisingly similar. For subsidies of small size, the welfare costs tend to be small but they become increasingly large as the subsidies increase in cost.

This result may be unexpected. But it is in line with the intuition for the results in table 6 that the method of finance was of secondary importance for a net oil importer. The biggest driver of the welfare losses is the distortion in relative prices that occurs with the subsidies, not the tax instrument used to finance it. This distortion is a feature of the subsidy whether the country is a net oil exporter or importer. Consequently the aggregate welfare results are similar in both cases.

6 Sensitivity Analysis

As a robustness check sensitivity analysis is performed on several of the model's parameters and variables. For brevity's sake only the results for the net oil importing country where both households and firms receive the subsidy are presented.²¹

6.1 Low Price Elasticities

In the baseline calibration the elasticities of substitution for oil products are set at a level consistent with long-run price elasticities. However, there is some variation in the estimates of long-run elasticities and short-run elasticities tend to be much smaller. In this section the previous exercises for the net oil importer are repeated using a new calibration where σ_c , σ_T , and σ_n are set to 0.25. This is consistent with a smaller price elasticity of demand.

Table 8 show the results. Lowering the price elasticity does have quantitative implications for the aggregate welfare results and the changes in the

²⁰One might be concerned that the assumption of a fixed world price of oil would be violated in this case as the country's exports change due to the subsidy. However, for any one individual country, the increase in exports would be fairly small. For example, Ecuador's entire domestic oil consumption in 2011 was only about .2 percent of total world demand. The increase in exports that would occur by removing the subsidies would only be a fraction of the total demand.

²¹Results for other cases are available upon request.

variables across steady states. Holding subsidy costs constant, the aggregate welfare costs are slightly reduced when compared to the baseline calibration. This is despite the fact that P^s drops by a greater amount when the elasticities are lower. The intuition for this is that with low price-elasticities both households and firms are much less responsive to the distorted relative price. Consequently consumption of oil products increases at a much lower pace than what occurs in the baseline model, and changes in other variables due to the distorted price are lower. This produces lower welfare losses across the board. This alternative calibration does not change the result that the method of financing the subsidy is of secondary importance to the results.

6.2 Higher use of Oil

Firm and household use of oil relative to the economy's GDP was calibrated using average values found in data on consumption expenditure shares and on firm spending on oil products found in input-output tables. Across countries there is a good deal of variation in those series, with some countries having higher shares than others. To consider the importance of this an alternative calibration is considered here where total firm spending on oil and the consumption expenditure share is doubled from the original calibration.

The results for this case are listed in table 9. Higher usage of oil in the economy increases the base that is being subsidized which means for a given subsidy cost the change in P^s is less than what occurs in the baseline calibration. Because of this, the welfare losses are smaller for a given subsidy size. Changes in the variables across steady states are qualitatively similar to the baseline calibration.

6.3 Asymmetric Taxation

The baseline model assumes that the traded and non-traded sectors are taxed at equal rates. However, this may not be the case for several reasons. First, in many developing countries there are excise taxes and import duties which get applied to imported goods. This might lead to consumption taxes falling more heavily on traded goods instead of non-traded goods. Second, the economies considered here often have large informal sectors, in the sense that portions of the economy avoid being taxed by the government. If one assumes that this is generally the non-traded services sector, then labor taxes might fall more heavily on employment in the traded sector. Both of these considerations can be handled in a relatively simple way in the model by assuming that different tax rates apply to the traded and non-

traded consumption goods and to labor income earned in the traded and non-traded sectors.

To consider the importance of these issues I repeat the previous exercises but consider extreme cases where the non-traded sector completely avoids taxation. In other words, if the consumption tax is used to finance the subsidy, then only traded consumption goods get taxed. If labor taxes are used, then only income in the traded sector gets taxed.

Table 10 show the results for this case. In regards to aggregate welfare the results are fairly similar to the baseline calibration. There are some differences in the responses of the variables compared to the baseline situation. However, these have less to do with the subsidy and more to do with the fact that the asymmetric taxation opens gaps between the wages and relative prices in the traded and non-traded sectors. For example, if income from the traded sector is taxed but income from the non-traded sector is not, then on the margin households choose to work a little more in the non-traded sector and a little less in the traded sector. This generates a gap between the wages in the formal and informal sectors (here the non-traded sector).²² This additional distortion affects aggregate welfare but not enough to overcome the large effects being driven by the subsidy itself.

6.4 High Frisch Elasticity of Labor Supply

The baseline calibration of μ was set to 1. As discussed earlier, there is a good deal of uncertainty regarding the calibration of this parameter and macro estimates are often larger than 1. An alternative calibration of the Frisch elasticity of labor supply to 3 is considered here. The results are contained in table 11. The alternative calibration does not significantly change the aggregate welfare results from the baseline calibration. In terms of the variables, the higher elasticity puts a slightly greater emphasis on increasing hours worked vis-a-vis reducing consumption, leading to marginally higher increases in labor supplied and marginally smaller decreases in the consumption variables.

7 Conclusion

This paper has considered the impact that fuel subsidies have on macroeconomic variables and aggregate welfare in both net oil importing and export-

²²Similar results can be found in papers that deal explicitly with the informal sector, such as Ihrig and Moe (2004) and references therein.

ing countries. There were several important results for the net oil importing case. First, the subsidy reduces aggregate welfare. The losses are fairly small for subsidy sizes on the order of 1 percent of GDP, but grow quickly as the subsidy become more costly. Second, the bulk of the welfare losses are due to the distortions in relative prices from the subsidy, and not the method of financing the subsidy. Third, replacing subsidies with lump-sum transfers of equal value is a significantly better policy option because it eliminates the distorted relative price and increases aggregate welfare by a large amount. Finally, the subsidy has a number of unintended consequences on other macroeconomic variables. These include the possibility of crowding out non-oil consumption, distorting inter-sectoral labor allocations, and distorting the relative price of non-tradables to tradeables.

The case of a net oil exporter was also considered. In this model the government provides the subsidy by selling a portion of its oil endowment below the world price of oil. Despite financing the subsidy in a different way, the implications of the subsidy on aggregate welfare for the net oil exporter turn out to be surprisingly similar to the case of the net oil importer. This is because the distorted relative price introduced by the subsidy is responsible for the bulk of the welfare losses.

This work considers the long-run implications of a particular type of subsidy that reduces the price of fuel in the steady state. There are several important directions on which this research could be expanded. First, all of the results here suppose that the country in question is small which implies that the subsidy does not influence the world price of oil. It would be interesting to consider the implications these subsidies (and their removal) might have on the world economy if the bloc of subsidizing countries changed their policies as a group. Distributional issues about the subsidies and the taxes that finance them have also been ignored in this paper but may be important. Such an approach would also be beneficial as it could ask about increasing transfers that target particular groups of households. Finally, this paper does not make any comments on the political economy aspects of fuel subsidies and why governments choose to have them. All of these areas are potentially fruitful avenues for future research on fuel subsidies.

Figure 1: The Size of Oil Subsidies Across Countries in 2011

Sources: IEA data, IMF World Economic Outlooks, author's calculations.

Figure 2: Welfare Costs as a Function of P^s

Table 1: Top Five Fuel Subsidizers in 2011 (IEA Data)

By Dollar Value				
Net Importers			Net Exporters	
Rank	Country	Total Subsidies (\$B)	Country	Total Subsidies (\$B)
1	India	30.9	Saudi Arabia	46.1
2	China	18.5	Iran	41.4
3	Indonesia	15.7	Venezuela	22.0
4	Egypt	15.3	Iraq	20.4
5	Thailand	3.3	Mexico	15.9

By Share of GDP				
Net Importers			Net Exporters	
Rank	Country	Share of GDP	Country	Share of GDP
1	Egypt	6.5	Iraq	17.8
2	Indonesia	1.9	Iran	8.6
3	India	1.7	Ecuador	8.2
4	Pakistan	1.3	Saudi Arabia	7.7
5	Sri Lanka	1.4	Libya	6.3

Sources: IEA, IMF WEOs, author's calculations.

Table 2: Subsidy Costs in Country Examples (Percent of GDP)

	Indonesia	Lebanon	Ukraine
Year	Oil	Electricity	Natural Gas
2000	5.4	1.1	N\A
2001	4.6	1.1	N\A
2002	1.9	1.0	N\A
2003	1.5	1.4	N\A
2004	3	2.1	N\A
2005	3.4	3.0	N\A
2006	1.9	3.4	N\A
2007	2.2	3.1	3.4
2008	2.8	5.1	4.5
2009	0.8	4.1	4.5
2010	1.3	3.0	3.8
2011	2.2	4.5	4.0
2012 (est.)	2.4	4.5	N\A

Sources: IEA, IISD (2012), IMF CRs, IMF WEOs, World Bank (2008), author's calculations.

Parameter or variable	Table 3: Calibration	
	Value	Source
Elasticity of intertemporal substitution (τ)	.50	Table 10.1 in Agenor and Montiel (1996)
Elasticities of substitution ($\sigma_c, \sigma_T, \sigma_n$)	.75, .25	Graham and Glaister (2002)
Frisch elasticity of labor supply (μ)	1, 3	See discussion in paper.
Time preference rate (ρ)	.06	Real interest rates in LDCs.
Consumption-expenditure share of oil	3%	Estimates from 19 developing countries.
Share of non-tradables in total consumption	50%	Buffie et al. (2008)
Total firm demand for oil	5% of GDP	Input-output tables for 11 developing countries.
Share of oil used by traded sector	40% of total	Input-output tables for 11 developing countries.

Table 4: Results for Net Oil Importer
Aggregate Welfare Costs of Subsidy

Case 1: Households Receive Subsidy				
Subsidy Financed by				
Subsidy Cost (% of GDP)	Lump-Sum Tax	Labor Tax	Consumption Tax	
1 %	.11	.11	.12	
2 %	.40	.40	.42	
3 %	.82	.82	.86	
4 %	1.34	1.34	1.40	
Case 2: Firms Receive Subsidy				
Subsidy Financed by				
Subsidy Cost (% of GDP)	Lump-Sum Tax	Labor Tax	Consumption Tax	
1 %	.07	.07	.07	
2 %	.27	.27	.27	
3 %	.57	.56	.56	
4 %	.94	.94	.94	
Case 3: Household and Firms Receive Subsidy				
Subsidy Financed by				
Subsidy Cost (% of GDP)	Lump-Sum Tax	Labor Tax	Consumption Tax	
1 %	.05	.04	.05	
2 %	.18	.17	.18	
3 %	.38	.37	.39	
4 %	.65	.64	.67	

Table 5: Results for Net Oil Importer
Changes in Macroeconomic Variables

Case 1: Households Receive Subsidy			
Subsidy Financed by			
Variable	Lump-Sum Tax	Labor Tax	Consumption Tax
P^s	-26.6	-26.6	-26.5
P^n	0	0	0
C^T	-0.4	-0.7	-0.8
C^n	-0.4	-0.7	-0.8
O^h	25.5	25.2	25.9
W^T	0	0	0
L	0.4	0.04	0.04
L^T	1.1	0.7	0.7
L^n	-0.4	-0.7	-0.8
Q^T	1.1	0.7	0.7
Q^n	-0.4	-0.7	-0.8
Case2: Firms Receive Subsidy			
Subsidy Financed by			
Variable	Lump-Sum Tax	Labor Tax	Consumption Tax
P^s	-17.2	-17.3	-17.3
P^n	-0.5	-0.5	-0.5
C^T	0.1	-0.2	-0.2
C^n	0.5	0.1	0.1
O^h	0.1	-0.2	0.5
W^T	0.7	0.7	0.7
L	0.4	0.02	0.02
L^T	1	0.7	0.7
L^n	-0.4	-0.7	-0.8
Q^T	1.6	1.2	1.3
Q^n	0.5	0.1	0.1
Case 3: Households and Firms Receive Subsidy			
Subsidy Financed by			
Variable	Lump-Sum Tax	Labor Tax	Consumption Tax
P^s	-11.4	-11.4	-11.4
P^n	-0.3	-0.3	-0.3
C^T	-0.1	-0.4	-0.4
C^n	0.1	-0.2	-0.2
O^h	9.4	9.1	9.9
W^T	0.5	0.5	0.5
L	0.3	0.01	0.02
L^T	1	0.7	0.7
L^n	-0.4	-0.7	-0.8
Q^T	1.4	1	1
Q^n	0.1	-0.2	-0.2

Table 6: Subsidies vs. Transfers in the Net Oil Importing Case
Households and Firms Receive Subsidy

Cost (% of GDP)	Labor Tax		Consumption Tax	
	Transfers	Subsidy	Transfers	Subsidy
1	.002	.04	.002	.05
2	.007	.17	.007	.18
3	.016	.37	.016	.39
4	.028	.67	.028	.67

Table 7: Aggregate Welfare Costs for the Net Oil Exporter
Households and Firms Receive Subsidy

Subsidy Cost (% of GDP)	Aggregate Welfare Cost
2 %	.20
4 %	.70
6 %	1.42
8 %	2.28

Table 8: Results for Low Price-Elasticity Case (Net Oil Importer)

Households and Firms Receive Subsidy

Aggregate Welfare Costs of Subsidy			
Subsidy Financed by			
Subsidy Cost (% of GDP)	Lump-Sum Tax	Labor Tax	Consumption Tax
1 %	.04	.04	.04
2 %	.15	.15	.15
3 %	.34	.33	.34
4 %	.59	.58	.58

Changes in Macroeconomic Variables

Subsidy Financed by

Variable	Lump-Sum Tax	Labor Tax	Consumption Tax
P^s	-12.1	-12.1	-12.1
P^n	-0.3	-0.3	-0.3
C^T	0.2	-0.1	-0.1
C^n	0.3	-0.1	-0.1
O^h	3.5	3.1	3.4
W^T	0.5	0.5	0.5
L	0.3	0.01	0.01
L^T	0.6	0.2	0.2
L^n	0.1	-0.3	-0.3
Q^T	0.7	0.4	0.4
Q^n	0.3	-0.1	-0.1

Table 9: Results for High Oil Use Case (Net Oil Importer)
Households and Firms Receive Subsidy

Aggregate Welfare Costs of Subsidy			
Subsidy Financed by			
Subsidy Cost (% of GDP)	Lump-Sum Tax	Labor Tax	Consumption Tax
1 %	.02	.02	.02
2 %	.09	.09	.09
3 %	.21	.20	.20
4 %	.36	.34	.34

Changes in Macroeconomic Variables			
Subsidy Financed by			
Variable	Lump-Sum Tax	Labor Tax	Consumption Tax
P^s	-6	-6	-6
P^n	-0.3	-0.3	-0.3
C^T	-0.1	-0.4	-0.5
C^n	0.2	-0.2	-0.2
O^h	4.6	4.3	5.1
W^T	0.4	0.4	0.4
L	0.3	0.01	0.01
L^T	0.9	0.6	0.6
L^n	-0.4	-0.7	-0.8
Q^T	1.3	0.9	1
Q^n	0.2	-0.2	-0.2

Table 10: Results under Asymmetric Taxation (Net Oil Importer)

Households and Firms Receive Subsidy		
Aggregate Welfare Costs of Subsidy		
Subsidy Cost (% of GDP)	Subsidy Financed by	
	Labor Tax	Consumption Tax
1 %	.04	.05
2 %	.16	.20
3 %	.36	.44
4 %	.61	.75

Changes in Macroeconomic Variables

Variable	Subsidy Financed by	
	Labor Tax	Consumption Tax
P^s	-11.5	-11.4
P^n	-2.1	-0.3
C^T	-1.1	-1.2
C^n	0.5	0.6
O^h	8.4	9.9
W^T	0.5	0.4
W^n	-1.4	0.4
L	0.01	0.02
L^T	0	0.03
L^n	0	0
Q^T	0.4	0.4
Q^n	0.5	0.6

Table 11: Results for High Frisch Elasticity Case (Net Oil Importer)
Households and Firms Receive Subsidy

Aggregate Welfare Costs of Subsidy				
Subsidy Financed by				
Subsidy Cost (% of GDP)	Lump-Sum Tax	Labor Tax	Consumption Tax	
1 %	.05	.04	.05	
2 %	.18	.17	.18	
3 %	.38	.37	.39	
4 %	.65	.64	.67	

Changes in Macroeconomic Variables			
Subsidy Financed by			
Variable	Lump-Sum Tax	Labor Tax	Consumption Tax
P^s	-11.4	-11.4	-11.4
P^n	-0.3	-0.3	-0.3
C^T	0	-0.4	-0.4
C^n	0.2	-0.2	-0.2
O^h	9.5	9.1	9.9
W^T	0.4	0.5	0.5
L	0.4	0.02	0.02
L^T	1.1	0.7	0.7
L^n	-0.3	-0.7	-0.8
Q^T	1.5	1	1.1
Q^n	0.2	-0.2	-0.2

References

- Agénor, P., Montiel, P. J., 1996. *Development Macroeconomics*. Princeton University Press.
- Aissa, M. S. B., Rebei, N., 2012. Price subsidies and the conduct of monetary policy. *Journal of Macroeconomics* 34, 769–787.
- Anand, R., Prasad, E. S., 2010. Optimal price indices for targeting inflation under incomplete markets, NBER Working Paper 16290.
- Bacon, R., Bhattacharya, S., Kojima, M., 2010. Expenditure of low-income households on energy. *Extractive Industries for Development Series No. 16*.
- Baig, T., Mati, A., Coady, D., Ntamungiro, J., 2007. Domestic petroleum product prices and subsidies: Recent developments and reform strategies, IMF Working Paper, WP/07/71.
- Buffie, E., Adam, C., O’Connell, S., Pattillo, C., 2008. Riding the wave: Monetary responses to aid surges in low-income countries. *European Economic Review* 52, 1378–1395.
- Catao, L., Chang, R., 2010. World food prices and monetary policy, NBER Working Paper 16563.
- Clements, B., Jung, H.-S., Gupta, S., 2003. IMF Working Paper 03/204.
- Coady, D., El-Said, M., Gillingham, R., Kpodar, K., Medas, P., Newhouse, D., 2006. The magnitude and distribution of fuel subsidies: Evidence from bolivia, ghana, jordan, mali, and sri lanka, IMF Working Paper, WP/06/247.
- Coady, D., Gillingham, R., Ossowski, R., Piotrowski, J., Tareq, S., Tyson, J., 2010. Petroleum product subsidies: Costly, inequitable, and rising, IMF Staff Position Note, SPN/10/05.
- EIA, 2011. Country analysis brief: Indonesia.
- EIA, 2012. International energy statistics.
- GIZ, 2010. International fuel prices survey.
- Graham, D. J., Glaister, S., 2002. The demand for automobile fuel: A survey of elasticities. *Journal of Transport Economics and Policy* 36, 1–26.

- IEA, 2010. World energy outlook 2010.
- IEA, 2011. World energy outlook 2011.
- IEA, 2012. Energy policies beyond ieA countries: Ukraine 2012.
- Ihrig, J., Moe, K., 2004. Lurking in the shadows: the informal sector and government policy. *Journal of Development Economics* 73, 541–557.
- IISD, 2012a. A citizen’s guide to energy subsidies in indonesia: 2012 update, Interntional Institute for Sustainable Development publication.
- IISD, 2012b. Indonesia’s fuel subsidies: Action plan for reform, Interntional Institute for Sustainable Development publication.
- IMF, 2012. Lebanon country report, cr 12/40.
- Kpodar, K., 2006. Distributional effects of oil price changes on household expenditures: Evidence from mali, IMF Working Paper, WP 06/91.
- Larsen, B., Shah, A., 1992. World fossil fuel subsidies and global carbon emissions, World Bank Policy Research Working Paper 1002.
- Mitra, P., Atoyan, R., 2012. Ukraine gas policy: Distributional consequences of tariff increases, IMF Working Paper 12/247.
- OECD, 2010. Phasing out energy subsidies, Article in OECD Economic Surveys: Indonesia 2010.
- Petri, M., Taube, G., Tsyvinski, A., 2002. Energy sector quasi-fiscal activities in the countries of the former soviet union, IMF Working Paper 02/60.
- Reichling, F., Whalen, C., 2012. Review of estimates of the frisch elasticity of labor supplyCongresson Budget Office Working Paper 2012-13.
- Vagliasindi, M., 2012. Implementing Energy Subsidy Reforms: Evidence from Developing Countries. World Bank.
- World Bank, 2007. Program document for a proposed energy development policy loan to the kingdom of morocco.
- World Bank, 2008. Republic of lebanon electricity sector public expenditure review.
- World Bank, 2010. Honduras: Power sector issues and options, ESMAP Formal Report 333/10.

A Model equations

$$\left(C^T^{\frac{\sigma_c-1}{\sigma_c}} + a_1 C^n^{\frac{\sigma_c-1}{\sigma_c}} + a_2 O^h^{\frac{\sigma_c-1}{\sigma_c}} \right)^{\left(\frac{\sigma_c}{\sigma_c-1} \right) \left(1 - \frac{1}{\tau} \right) - 1} C^{T-\frac{1}{\sigma_c}} = (1 + \tau^c) \lambda \quad (\text{A-1})$$

$$\left(C^T^{\frac{\sigma_c-1}{\sigma_c}} + a_1 C^n^{\frac{\sigma_c-1}{\sigma_c}} + a_2 O^h^{\frac{\sigma_c-1}{\sigma_c}} \right)^{\left(\frac{\sigma_c}{\sigma_c-1} \right) \left(1 - \frac{1}{\tau} \right) - 1} a_1 C^{n-\frac{1}{\sigma_c}} = (1 + \tau^c) P^n \lambda \quad (\text{A-2})$$

$$\left(C^T^{\frac{\sigma_c-1}{\sigma_c}} + a_1 C^n^{\frac{\sigma_c-1}{\sigma_c}} + a_2 O^h^{\frac{\sigma_c-1}{\sigma_c}} \right)^{\left(\frac{\sigma_c}{\sigma_c-1} \right) \left(1 - \frac{1}{\tau} \right) - 1} a_2 O^{h-\frac{1}{\sigma_c}} = P^s \lambda \quad (\text{A-3})$$

$$\kappa L^{\frac{1}{\mu}} = (1 - \tau^l) W^T \lambda \quad (\text{A-4})$$

$$\kappa L^{\frac{1}{\mu}} = (1 - \tau^l) W^n \lambda \quad (\text{A-5})$$

$$\frac{\dot{\lambda}}{\lambda} = \rho - r \quad (\text{A-6})$$

$$(1 - \tau^l) (W^n L^n + W^T L^T) + T = (1 + \tau^c) (C^T - P^n C^n) + P^s O^h \quad (\text{A-7})$$

$$Q^T = \left[(A^T L^T)^{\frac{\sigma_T-1}{\sigma_T}} + b^T (O^T)^{\frac{\sigma_T-1}{\sigma_T}} \right]^{\frac{\sigma_T}{\sigma_T-1}} \quad (\text{A-8})$$

$$Q^{T\frac{1}{\sigma_T}} (A^T L^T)^{-\frac{1}{\sigma_T}} A^T = W^T \quad (\text{A-9})$$

$$Q^{T\frac{1}{\sigma_T}} b^T (O^T)^{-\frac{1}{\sigma_T}} = P^s \quad (\text{A-10})$$

$$\Phi^T = \left[\left(\frac{W^T}{A^T} \right)^{1-\sigma_T} + b^{T\sigma_T} (P^s)^{1-\sigma_T} \right]^{\frac{1}{1-\sigma_T}} \quad (\text{A-11})$$

$$Q^n = \left[(A^n L^n)^{\frac{\sigma_n-1}{\sigma_n}} + b^n (O^n)^{\frac{\sigma_n-1}{\sigma_n}} \right]^{\frac{\sigma_n}{\sigma_n-1}} \quad (\text{A-12})$$

$$Q^{n\frac{1}{\sigma_n}} (A^n L^n)^{-\frac{1}{\sigma_n}} A^n = \frac{W^n}{P^n} \quad (\text{A-13})$$

$$Q^{n\frac{1}{\sigma_n}} b^n (O^n)^{-\frac{1}{\sigma_n}} = \frac{P^s}{P^n} \quad (\text{A-14})$$

$$\Phi^n = \left[\left(\frac{W^n}{A^n} \right)^{1-\sigma_n} + b^{n\sigma_n} (P^s)^{1-\sigma_n} \right]^{\frac{1}{1-\sigma_n}} \quad (\text{A-15})$$

$$C^n = Q^n \quad (\text{A-16})$$

$$b = 0 \quad (\text{A-17})$$

Government budget constraint for net oil importer

$$\tau^l (W^T L^T + W^n L^n) + \tau^c (C^T + P^n C^n) + T = Tr + (P^o - P^s) (O^h + O^T + O^n) \quad (\text{A-18})$$

Current account equation for net oil importer

$$Q^T = C^T + P^o (O^h + O^T + O^n) \quad (\text{A-19})$$

Government budget constraint for net exporter

$$P^o (Q^o - O^h - O^T - O^n) + P^s (O^h + O^T + O^n) = Tr \quad (\text{A-20})$$

Current account equation for net exporter

$$Q^T + P^o Q^o = C^T + P^o (O^h + O^T + O^n) \quad (\text{A-21})$$

B Data used for calibration

B.1 Consumption expenditure shares

Estimates were found for 18 developing countries. The average value of the 18 countries was 3.15 percent for oil products. This was rounded to 3 in the model calibration. Table B-1 provides the estimates and the sources for those estimates.

B.2 Firm use of oil

The OECD STAN database contains input-output tables for 11 developing countries. These tables provide data on spending by firms on “Coke, refined petroleum products and nuclear fuel” which provides a rough estimate of total firm spending on oil products. For the countries considered, the average was 5.3 percent of GDP with about 36 percent of that spending due to firms in the traded sector (defined as agriculture, mining, and manufacturing). These were rounded to 5 percent and 40 percent in the model calibration. The input-output tables also include spending on “Electricity, gas, and water supply.” Including this increases the total average to 9.9 percent of GDP with roughly 39 percent due to firms in the traded sector. These were rounded to 10 percent and 40 percent in the alternative calibration. Table B-2 lists the GDP ratios and the share that the traded sector makes out of the total. All calculations were done by the author.

C Additional countries and sources

C.1 Fuel subsidies

The IEA data lists 34 countries that had subsidies on fuel products at some point in time between 2007 and 2011. Using additional sources this list can

Table B-1: Consumption-Expenditure Shares

Country	Oil products	Electricity	Source
Bolivia	3.1	-	Coady et al. (2006)
Ghana	4.2	-	Coady et al. (2006)
Jordan	4.3	2.3	Coady et al. (2006)
Mali	2.9	0.4	Coady et al. (2006)
Sri Lanka	2.0	1.3	Coady et al. (2006)
Morocco	3.8	2.1	World Bank (2007)
Gabon	2.7	-	Bacon et al. (2010)
Madagascar	2.1	0.5	Bacon et al. (2010)
Bangladesh	1.4	1.1	Bacon et al. (2010)
Cambodia	1.2	0.8	Bacon et al. (2010)
India	3.4	2.4	Bacon et al. (2010)
Indonesia	3.8	3.4	Bacon et al. (2010)
Kenya	2.5	0.2	Bacon et al. (2010)
Pakistan	2.1	3.5	Bacon et al. (2010)
Thailand	6.7	3.1	Bacon et al. (2010)
Uganda	1.7	0.4	Bacon et al. (2010)
Vietnam	5.9	3.0	Bacon et al. (2010)
Burkina Faso	3.0	-	IMF CR 09/38

Table B-2: Energy-GDP ratios

Country	Oil	Traded sector share	Electricity	Traded sector share
Argentina	1.4%	35.7%	2.8%	32.1%
Brazil	4.7%	46.8%	5.4%	40.7%
Chile	4.3%	27.9%	5.3%	32.1%
China	7.2%	44.4%	9.0%	68.9%
India	7.1%	26.8%	4.5%	44.4%
Indonesia	6.4%	23.4%	2.5%	40.0%
Mexico	2.2%	27.3%	2.1%	33.3%
South Africa	9.7%	50.5%	2.6%	34.6%
Thailand	6.8%	32.4%	7.0%	44.3%
Turkey	3.1%	35.5%	5.6%	26.8%
Vietnam	5.6%	42.9%	3.5%	68.6%

be expanded to include additional countries and to consider years prior to 2007. In some cases quantitative data is also available regarding the size of a subsidy in a given year for a country. Table C-1 lists 21 additional countries identified as having a fuel subsidy sometime between 2000 and 2012.

C.2 Electricity subsidies

Additional sources also allow one to expand the list of countries with subsidies on electricity produced using fossil fuels. Table C-2 lists an additional 15 countries that had these subsidies at least one point in time between 2000 to 2012.

C.3 Data

Table C-4 provides the fuel subsidy data from the IEA for the 34 countries they identified for the years from 2007 to 2011. The dollar value has been converted into a subsidy-GDP ratio using GDP data from IMF World Economic Outlook reports.

Table C-5 provides additional data on costs of fuel subsidies and electricity subsidies, as a share of GDP. The numbers come from a variety of sources that have used different methods and data to calculate the costs of the subsidies. For this reason comparison across countries or with the IEA data should be done with extreme caution. Note that there are a number of gaps in the data. A blank entry does not mean the subsidy was non-existent. It merely means that no data was available for that year.

C.4 Sources

I relied heavily on Vagliasindi (2012), the GIZ International Fuel Prices surveys, and IMF country reports to identify the additional countries.

Maria Vagliasindi's book "Implementing Energy Subsidy Reforms: Evidence from Developing Countries" provided information on energy subsidies in 17 countries. Several countries were discussed in the book that are not found in the IEA data. These included Ghana, Jordan, Morocco, the Dominican Republic, and Yemen.

The GIZ releases an International Fuel Prices survey every two years. The latest edition is GIZ (2010). These surveys provide a data point for retail gasoline and diesel prices in a number of countries in November every two years. The time series for some countries goes back to 1991 but in most cases does not begin regularly until 1998. The following countries were

Table C-1: Additional Countries with Fuel Subsidies

Bahrain	Gabon	Oman
Bolivia	Ghana	Panama
Burkina Faso	Jordan	Senegal
Congo	Lebanon	Sudan
Dominican Republic	Mauritania	Syria
El Salvador	Morocco	Tunisia
Ethiopia	Nepal	Yemen

Table C-2: Additional Countries with Electricity Subsidies

Bolivia	Honduras	Mauritania
Cape Verde	Jordan	Mauritius
Djibouti	Lebanon	Nicaragua
Dominican Republic	Maldives	Panama
El Salvador	Mali	Senegal
Ghana		

identified using the GIZ International Fuel Prices Survey: Bahrain, Congo, Oman, and Syria.

IMF country reports and working papers identified the rest of the countries. Table C-3 lists the IMF reports used as sources. Country reports are shortened to CR and working papers to WP. The first column identifies the country, the second column the products discussed in one or more of the reports, and the third column provides a list of the documents. In the second column O is short-hand for oil products and E for electricity.

Please note that the list in the table is not exhaustive. For example, the IMF has country reports on the Dominican Republic which discuss electricity subsidies in that country. I omitted those country reports from the table since this country was already identified using Vagliasindi (2012). Reports which provided data on costs (which were not found elsewhere) are listed in the table.

The data on the costs of electricity subsidies in Honduras came from World Bank (2010).

Table C-3: IMF Sources		
Country	Products	Reports
Bolivia	E, O	CR 12/149
Burkina Faso	O	CR 06/359, WP 11/202
Cape Verde	E	CR 10/349, 11/254
Djibouti	E	CR 09/216, 10/277, 12/197
El Salvador	E, O	CR 09/35, 10/307
Ethiopia	O	CR 06/159, 08/259, 08/264, 09/34
Gabon	O	CR 06/238, 08/24, 09/107, 13/55, WP 06/243
Honduras	E	WP 08/168
Jordan	E, O	CR 12/119, 12/120
Lebanon	E, O	CR 04/313, 06/201, 07/382, 09/131, 10/306, 12/39, 12/40
Maldives	E	CR 09/97, 10/28, 10/167, 11/293
Mali	E	CR 11/141, 12/3
Mauritania	E, O	CR 11/362
Mauritius	E	CR 05/281, 08/237
Nepal	O	CR 07/204, 07/366, 08/181, 10/185, 12/326
Nicaragua	E	CR 06/174, 10/376, 11/118, 11/322, 12/256, 12/257
Panama	E, O	CR 06/130, 09/207, 12/83
Senegal	E, O	CR 07/335, 08/221, 09/5, 10/363, 12/337
Tunisia	O	CR 06/207, 08/345, 09/329, 10/282, 12/255

Table C-4: Fuel Subsidies as a Percent of GDP (IEA data)

Country	2007	2008	2009	2010	2011
Algeria	3.0	3.8	2.8	5.2	5.7
Angola	0.9	1.2	0.3	1.1	1.0
Argentina	1.5	2.0	0.2	0.2	0.4
Azerbaijan	0.5	0.7	0.0	0.2	1.0
Bangladesh	0.5	0.7	0.0	0.3	0.8
Brunei	1.4	1.8	1.0	1.5	1.9
China	0.3	0.6	0.1	0.1	0.3
Colombia	0.3	0.4	0.1	0.2	0.2
Ecuador	6.8	8.0	3.1	6.4	8.2
Egypt	10.7	12.1	5.2	6.4	6.5
India	1.5	2.5	0.9	1.0	1.7
Indonesia	2.6	2.8	1.7	1.4	1.9
Iran	11.9	15.3	8.1	9.8	8.6
Iraq	14.5	14.6	7.7	17.2	17.8
Kazakhstan	1.3	1.2	0.4	1.4	1.7
Kuwait	2.4	2.8	1.8	2.3	2.7
Libya	3.7	3.6	2.7	4.3	6.3
Malaysia	1.4	2.0	0.8	1.6	1.9
Mexico	1.6	2.0	0.4	0.9	1.4
Nigeria	1.2	1.4	0.0	1.1	1.5
Pakistan	1.9	2.5	0.0	0.1	1.3
Peru	0.1	0.2	0.0	0.0	0.2
Philippines	0.1	0.1	0.0	0.6	0.6
Qatar	1.2	1.3	0.7	0.9	1.2
Saudi Arabia	6.2	7.6	5.9	6.7	7.7
South Africa	0.1	0.1	0.0	0.0	0.0
Sri Lanka	1.2	1.6	0.0	0.6	1.4
Taiwan	0.2	0.3	0.1	0.1	0.1
Thailand	0.6	0.8	0.5	0.7	1.0
Turkmenistan	2.8	3.3	3.7	3.9	3.0
UAE	0.6	0.6	0.4	0.9	1.1
Uzbekistan	0.0	0.0	1.3	0.7	2.3
Venezuela	5.7	5.2	3.2	5.3	6.9
Vietnam	0.4	1.2	0.0	0.0	0.8

Sources: IEA, IMF WEOs, author's calculations.

Table C-5: Additional Data on Subsidy Costs

	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012
Fuel Subsidies													
Bolivia							0.5	0.9	2	1.3	1.7	2.9	
Burkina Faso							0.7		0.8				
Dominican Republic							0.5	0.4	0.5	0	0	0	
Gabon				0.4	0.7	0.4	2	1	1.3				1.8
Ghana				2	2.2	1.7	1.5	0	0.1	0.3	0.3		
Indonesia	5.4	4.6	1.9	1.5	3	3.4	1.9	2.2	2.8	0.8	1.3	2.2	2.4
Jordan			0.6	1.2	3.2	5.6	2.8	2.7	1.2	0.2	0.4	2.4	1.3
Morocco			0	0.6	1.1	1.4	1.3	3.9	3.7	1.5	1.2		
Tunisia					0.6	1.1	1.2	1	1.5	0.7	0.9	2.4	2.4
Senegal				0.0	0.2	0.3	1.3	1.0	1.2	0.5	0.2	0.2	0.1
Yemen	5.7	3.8	3	4.8	5.7	8.7	8.1	9.3	14.5	9.5	8.2		
Electricity Subsidies													
Dominican Republic					2	1.7	1.4	1.2	2.7	1.4	1.2		
Honduras			2.6	2.4	2.1	1	1.3						
Indonesia					0.3	0.3	0.9	0.8	1.7	0.9	0.9	1.2	1.1
Jordan											0.9	5.4	
Lebanon	1.1	1.1	1.0	1.4	2.1	3.0	3.4	3.1	5.1	4.1	3.0	4.5	4.5
Mali											0.3	0.4	0.4
Nicaragua						0.6							0.6
Senegal						1.8	2.7	1.5	1.6	1.1	1.7	2.1	
Yemen	0	0	0	0.3	0.2	0.2	0.1	0.1	0.1	0.1	0.1		
Only Total Available													
El Salvador									2.5	1.4			
Mauritania									0.9	0.8	0	1.6	2.4

Sources: Refer to appendix.