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The unbalanced matching in a director market

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Abstract

I construct an intertemporal searching model ("take it or leave it offer") in a frictional directorship market to explain the unbalanced matching between the director and the firm. In this model, potential candidates for outside directors and firms have heterogeneous (also, well ordered) quality levels. Also, both parties have strictly ordered preferences over the quality of counterpart from high levels to low levels. A candidate considers his quality ranking to compare the value of accepting a favorite offer at present to the value of waiting for successful matching with a better offer in the future. My model suggests that that highly qualified candidates would be likely to be matched with bad (not too bad) firms. The best candidate could go to the 150th ranked firm over 250 firms under the uniform distribution for the quality of the firm, and the 140th ranked firm under the extreme value distribution.

JEL Classification: G34, G38, J41, J44, J64

Keywords: Corporate Governance, Board of Directors, Matching Model, Job Search

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1 Introduction

Gabaix and Landier (2006) propose a simple competitive assignment model to explain CEO compensation. They assume that CEOs have heterogeneous talent level and are assigned to firms competitively. The managerial impact of a CEO's talent increase with the value of the firm under his control. Under these assumptions, they find that the best CEO goes to the largest firms and a CEO's pay increases in the size of firm and size of average firm in the economy. Their empirical finding supports these predictions.

However, we can easily find the mismatch between the director and the firm in the directorship market. I analyze 250 U.S. firm's board profile among Fortune 500 firms in 2005. Then, I rank all ongoing and retired CEOs who are working as outside directors on the boards based on the size of firms (market capitalization) at which they are working or worked as CEO. The data shows that, for instance, the first ranked director is matched with 176th ranked firm, and the 6th ranked director is matched with 246th ranked firm in terms of the market capitalization in 2005.

To explain the unbalanced match, I construct an intertemporal searching and matching model ("take it or leave it offer") in a frictional directorship market under four main assumptions. First, the candidates for outside directors want to maximize their reputation value which depends on the size of the firm. Second, there is no wage competition or wage bargaining in a director market. It naturally follows that the firms simply prefer highly qualified directors. Third, outside director positions are randomly opened over the time frame. Finally, potential candidates for outside directors and firms have heterogeneous (also, well ordered) quality levels. Also, both parties have strictly ordered preferences over the quality of counterpart from high levels to low levels. A candidate considers his quality ranking to compare the value of accepting a favorite offer at present to the value of waiting for successful matching with a better offer in the future.

Under a certain assumption, there is the possibility that highly qualified candidates are matched with bad (not too bad) firms. The best candidate would be likely to go to the 150th ranked firm over 250 firms under the uniform distribution and go to 140th ranked firms under Gabaix and Landier (2006)'s extreme value distribution. Additionally, I find that the higher ranked (better talented) candidates have higher cutoff ranking of firms but the marginal increase in the ranking of firm is diminishing. For instance, if the firm quality follows an uniform distribution, (if candidates consider the ranking of firm, e.g, *Fortune 500*, as the quality of firm), the difference between the cutoff level of 1st ranked candidate and 20th ranked one is 10 in terms of firm ranking, but the difference between 20th ranked candidate and 30 ranked one is 64. If the firm quality follows the distribution of CEO talent inferred by Gabaix and Landier (2006) based on the extreme value theory (if candidates consider the talent level of CEO as the quality of firm), the gap between 1st ranked and 20th ranked candidate is also 10 and between 20th and 30th is 68.

The rest of the paper is organized as follows. In section 2, I provide a brief review of the related literature. In section 3, I develop an intertemporal searching and matching

model. The Section 4 show the numerical analysis. I summarize concluding remarks in Section 5.

2 Related literature

2.1 The matching/searching and matching

Gale and Shapley (1962) study the classic matching problem in the marriage market. They assume that each part (man and woman) ranks counterparts based on idiosyncratic preferences and show that there always exists a stable assignment equilibrium (assortative matching). In contrast, this paper assumes that both parties have the same preference order. They simply prefer high quality to low quality. McCall (1970) develops the intertemporal searching and matching model in the labor market. He sets up the model of representative agent who lives forever and uses sequential search technique to explain the determination of reservation wage. I also analyze the determination of reservation quality level based on the intertemporal searching and matching framework, but both parties in the matching market are heterogenous and well-ranked in terms of quality. Gabaix and Landier (2006) propose a simple competitive assignment model to explain CEO compensation. They assume that CEOs have heterogenous talent level and are assigned to firms competitively. The managerial impact of a CEO's talent increase with the value of the firm under his control. Under these assumptions, they find that the best CEO goes to the largest firms and a CEO's pay increases in the size of firm and size of average firm in the economy. Their empirical finding supports these predictions. In this paper, I study the matching problem in a frictional director market. The vacancies for the outside directors are assumed to be randomly opened over time.

3 Model

I develop an intertemporal searching and matching model in which potential candidates for outside directorships live forever and are risk neutral. Based on Gale and Shapley (1962), McCall (1970) and Gabaix and Landier (2006) I construct the basic setting. There are m number of potential candidates for outside directorships with an well-ordered quality level, q_{new}^k . k denotes the ranking of his quality level in the pool of potential candidates. There is no possibility that $q_{new}^i = q_{new}^j$ when $i \neq j$. I assume that the number of potential candidates, m , is time-invariant on the steady-state, which means that the inflow into the potential candidates pool is equal to the outflow from the potential candidates pool. I also assume that each potential candidate's ranking of his quality is time-invariant on the steady state.

At each time, the n number of firms each creates one vacancy for an outside director position. The n number of vacancies for an outside director position are randomly created in a sense that the quality of firms, q_f^1 , is randomly drawn from a nondecreasing

¹We can interpret the quality of firm in three different ways: (1) the ranking of firm in the economy,

and continuous distribution $F(q_f)$ on $[0, \overline{q_f}]$. Also, $F(0) = 0$ and $F(\overline{q_f}) = 1$. The distribution of quality of firms is well dispersed, so that there is no possibility for $q_f^i = q_f^j$ when $i \neq j$. i and j denote the ranking of quality level of firm. All firms opening an outside directorship position have a list of potential candidates. It implies that they have informations for the well-ordered quality level of potential candidates for outside directors. All firms have a strictly ordered preference over the quality of candidate, say, $q_{new}^1 \succ q_{new}^2 \succ q_{new}^3 \dots$. Also, all candidates have a strictly ordered preference over the quality of firms, say, $q_f^1 \succ q_f^2 \succ q_f^3 \dots$.

The stage is for the following. At first, all firms offer to a favorite one (the first ranked candidate). The first ranked candidate rejects all but his favorite one and decide whether to accept this favorite offer or also reject this to allow for the possibility that the better offer may come along later. In the second stage, those firms who are rejected give offer to their second choice. The second ranked candidate iterates this process. If $m \leq n$ all potential candidates will eventually have received an offer and made decisions. If $m > n$ only the first n number of potential candidates will surely have received an offer and made decisions². I will below focus on the steady state equilibrium in the latter case³.

3.1 The value function

If a candidate with the quality ranking, k , has an favorite offer in the first n number of job opening is $q_{k,f}$, then he will make a decision whether to accept or wait for a better offer in the future. Neither quitting nor firing is allowed. At each period, a candidate chooses a strategy to maximize $E \sum_{t=0}^{\infty} (\frac{1}{1+r})^t y_t$, where y_t is the reputation value from working as outside director depending on $q_{k,f}$. The bellman equation for the candidate's problem is

$$V(q_{k,f}) = \max \left\{ V^a(q_{k,f}), \frac{1}{1+\gamma} EV(q'_{k,f}) \right\}$$

where $V(q_{k,f})$ denotes the value of having an offer, $q_{k,f}$, in hand and $V^a(q_{k,f})$ denotes the value of accepting a current offer. $q'_{k,f}$ is the offer in the next period. Then, the solution will be

$$V(q_{k,f}) = \begin{cases} \frac{y(q_{k,f}^{cut})}{\gamma} = \frac{1}{1+\gamma} E_{prob^k, q'_{k,f}} [V(q'_{k,f})] & \text{if } q_{k,f} \leq q_{k,f}^{cut} \\ \frac{y(q_{k,f})}{\gamma} & \text{if } q_{k,f} \geq q_{k,f}^{cut} \end{cases}$$

(2) the quality of CEO, and (3) the size of firm. The functional form of distribution $F(q_f)$, depends on interpretations.

²Suppose there are n number of job openings and $n + 1$ number of potential candidates. Then, the $n + 1^{th}$ ranked candidate will get an offer only if at least one of higher ranked candidates than him will reject an favorite offer. In general, we can not guarantee that all potential candidates will have received an offer.

³The reasons are that (1) this model can generally guarantee that the first n number of upper ranked candidates will surely have received an offer and made decisions and (2) I want to focus on the cutoff level of highly ranked candidates in the pool of potential candidates.

where $q_{k,f}^{cut}$ is the cutoff quality with the property that a candidate should accept an offer $q_{k,f} \geq q_{k,f}^{cut}$ and reject an offer $q_{k,f} \leq q_{k,f}^{cut}$. More specifically, the value function of accepting this offer is given by

$$V^a(q_{k,f}) = y(q_{k,f}) + \frac{1}{1+\gamma} V^a(q_{k,f})$$

The flow value for a candidate who will serve as an outside director on boards with quality level, $q_{k,f}$, denoted by $V^a(q_{k,f})$ equals the sum of the flow return, the reputation value from working as outside director, $y(q_{k,f})$, plus the discounted expected flow value defined by $\frac{1}{1+\gamma} V^a(q_{k,f})$. We can rewrite above equation by

$$V^a(q_{k,f}) = \frac{y(q_{k,f})}{\gamma} \quad (1)$$

where the reputation function, $y(q_{k,f})$, is increasing in $q_{k,f}$ and $y(0) = 0$. The value from rejection is defined by

$$V^r(k) = \frac{1}{1+\gamma} E_{prob^k, q'_{k,f}} [\max \{V^a(q'_{k,f}), V^r(k)\}]$$

The value for a candidate of quality ranking, k , who waits for better offer is denoted by $V^r(k)$ which equals the return from waiting and drawing again. Here, the expectation operation (E) is with respect to two components. One is the probability of successful matching with a better position, $prob^k$, and the other is the value of $q'_{k,f}$. In this model, all candidates have a well-ordered quality ranking and "take or leave it process" at each period mainly depends on their ranking, so that $prob^k$ plays a crucial role in determining the policy. More specifically, the value of rejection is given by

$$V^r(k) = \frac{1}{1+\gamma} \left\{ prob^k(q'_{k,f} \geq q_{k,f}^{cut}) E_{q'_{k,f}} [V^a(q'_{k,f})] + \left(1 - prob^k(q'_{k,f} \geq q_{k,f}^{cut})\right) V^r(k) \right\} \quad (2)$$

where the expectation is only with respect to the value of $q'_{k,f}$. The value for a candidate of quality ranking, k , who waits for better offer is denoted by $V^r(k)$ equals the discounted expected value from getting a better offer, $q'_{k,f} \geq q_{k,f}^{cut}$, in the future, which is defined by

$$\frac{1}{1+\gamma} prob^k(q'_{k,f} \geq q_{k,f}^{cut}) E_{q'_{k,f}} [V^a(q'_{k,f})]$$

and the discounted expected value from rejection, denoted by

$$\frac{1}{1+\gamma} \left(1 - prob^k(q'_{k,f} \geq q_{k,f}^{cut})\right) V^r(k)$$

$prob^k(q'_{k,f} \geq q_{k,f}^{cut})$ denotes the probability that the k th ranked candidate may get a better offer, $q'_{k,f} \geq q_{k,f}^{cut}$, in the next period. In other words, $prob^k(q'_{k,f} \geq q_{k,f}^{cut})$ represents the probability of successful matching with a better position than his cutoff quality. The

probability function has the following property.

$$prob^k(q'_{k,f} \geq q_{k,f}^{cut}) = \begin{cases} 0 & \text{if } q_{k,f}^{cut} = \bar{q}_f \\ 1 & \text{if } q_{k,f}^{cut} = 0 \end{cases}$$

We can rewrite the equation (2) by

$$\left(r + prob^k(q'_{k,f} \geq q_{k,f}^{cut})\right) V^r(k) = \left\{ prob^k(q'_{k,f} \geq q_{k,f}^{cut}) E_{q'_{k,f}}[V^a(q'_{k,f})] \right\}$$

Finally, we can get

$$V^r(k) = \frac{prob^k(q'_{k,f} \geq q_{k,f}^{cut}) E_{q'_{k,f}}[V^a(q'_{k,f})]}{\left(r + prob^k(q'_{k,f} \geq q_{k,f}^{cut})\right)} \quad (3)$$

3.2 The cutoff quality, $q_{k,f}^{cut}$

The solution will be

$$V(q_{k,f}) = \begin{cases} \frac{y(q_{k,f}^{res})}{\gamma} = \frac{1}{1+\gamma} E_{prob^k, q'_{k,f}}[V(q'_{k,f})] & \text{if } q_{k,f} \leq q_{k,f}^{cut} \\ \frac{y(q_{k,f})}{\gamma} & \text{if } q_{k,f} \geq q_{k,f}^{cut} \end{cases}$$

Henceforth, the endogenous cutoff quality level, denoted by $q_{k,f}^{cut}$ satisfies the following condition.

$$\begin{aligned} \frac{y(q_{k,f}^{cut})}{\gamma} &= \frac{1}{1+\gamma} E_{prob^k, q'_{k,f}}[V(q'_{k,f})] \\ ; \frac{y(q_{k,f}^{cut})}{\gamma} &= \frac{prob^k(q'_{k,f} \geq q_{k,f}^{cut}) E_{q'_{k,f}}[V^a(q'_{k,f})]}{\left(r + prob^k(q'_{k,f} \geq q_{k,f}^{cut})\right)} \end{aligned}$$

Rewriting above equation, we can get

$$y(q_{k,f}^{cut}) = \frac{\gamma}{\left(r + prob^k(q'_{k,f} \geq q_{k,f}^{cut})\right)} prob^k(q'_{k,f} \geq q_{k,f}^{cut}) E_{q'_{k,f}}[V^a(q'_{k,f})] \quad (4)$$

The left-hand side is the cost of searching one more time when he got an offer, $q_{k,f}^{cut}$ and the right-hand side is the expected benefit of searching one more time. The equation (3) makes the candidate to set $q_{k,f}^{cut}$ to equate the cost and benefit of searching one more time.

Proposition 1 *Let me define the right hand side of equation (3) as*

$$B(q_{k,f}^{cut}) \equiv \frac{\gamma}{\left(r + prob^k(q'_{k,f} \geq q_{k,f}^{cut})\right)} prob^k(q'_{k,f} \geq q_{k,f}^{cut}) E_{q'_{k,f}}[V^a(q'_{k,f})]$$

If $EV^a(k, 0, q_{k,f}^{high}) > 0$ and $\frac{\partial B(q_{k,f}^{cut})}{\partial q_{k,f}^{cut}} < 0$ on $(0, \bar{q}_f)$, there exists a unique cutoff level, $q_{k,f}^{cut}$, $0 < q_{k,f}^{cut} < \bar{q}_f$, which guarantees that the k th ranked candidate accept an offer, $q_{k,f}$, if $q_{k,f} \geq q_{k,f}^{cut}$. Otherwise, he waits for a better offer in the future

Proof. See Appendix ■

First, I focus on the cutoff level of the 1st ranked candidate, $q_{1,f}^{cut}$. If at least one job opening out of n number of job openings is drawn from the sapce above $q_{1,f}^{cut}$ the 1st ranked candidate gets a better offer. The probability that the 1st ranked candidate can get a better offer, $q_{1,f}^{cut} < q_{1,f}^{high} \leq \bar{q}_f$, is defined by

$$prob^1(q_{1,f}^{cut}) = \sum_{t=1}^n C_t^n (1 - F(q_{1,f}^{cut}))^t F(q_{1,f}^{cut})^{n-t} = 1 - F(q_{1,f}^{cut})^n$$

The corresponding expected value of better offer is

$$\int_{q_{1,f}^{cut}}^{\bar{q}_f} \frac{y(q_{1,f}^{high})}{\gamma} f(q_{1,f}^{high}) dq_{1,f}^{high}$$

Henceforth, his cutoff level, $q_{1,f}^{cut}$, solves to

$$\frac{y(q_{1,f}^{cut})}{\gamma} = \frac{\left(\sum_{t=1}^n C_t^n (1 - F(q_{1,f}^{cut}))^t F(q_{1,f}^{cut})^{n-t} \right) \int_{q_{1,f}^{cut}}^{\bar{q}_f} \frac{y(q_{1,f}^{high})}{\gamma} f(q_{1,f}^{high}) dq_{1,f}^{high}}{r + \left(\sum_{t=1}^n C_t^n (1 - F(q_{1,f}^{cut}))^t F(q_{1,f}^{cut})^{n-t} \right)}$$

Then, we can have the following proposition.

Proposition 2 *The probability of successful matching with a better position than $q_{1,f}^{cut}$, $prob^1(q_{1,f}^{cut})$, decreases in $q_{1,f}^{cut}$ and there exists a unique cutoff level, $q_{1,f}^{cut}$, which guarantees that the 1st ranked candidate accepts an offer, $q_{1,f}$, if $q_{1,f} \geq q_{1,f}^{cut}$. Otherwise, he waits for a better offer in the future.*

Proof. See Appendix ■

Similarly, the probability that a 2nd ranked candidate can get a better offer, $q_{2,f}^{cut} < q_{2,f}^{high} < \bar{q}_f$, is

$$prob^2(q_{2,f}^{cut}) = \left\{ \begin{array}{l} C_1^n \left\{ F(q_{1,f}^{cut}) - F(q_{2,f}^{cut}) \right\} F(q_{2,f}^{cut})^{n-1} \\ + \sum_{t=2}^n C_t^n (1 - F(q_{2,f}^{cut}))^t F(q_{2,f}^{cut})^{n-t} \text{ if } 0 \leq q_{2,f}^{cut} < q_{1,f}^{cut} \\ \sum_{t=2}^n C_t^n (1 - F(q_{2,f}^{cut}))^t F(q_{2,f}^{cut})^{n-t} \text{ if } q_{1,f}^{cut} \leq q_{2,f}^{cut} < \bar{q}_f \end{array} \right\}$$

Suppose that his cutoff level is below the cutoff level of the 1st ranked candidate. If only one job opening is drawn from the space between $q_{2,f}^{cut}$ and $q_{1,f}^{cut}$ he gets this offer because the 1st ranked candidate will reject this one. $C_1^n \left\{ F(q_{1,f}^{cut}) - F(q_{2,f}^{cut}) \right\} F(q_{2,f}^{cut})^{n-1}$ shows this probability. In this case, the expected value of better offer is given by

$$\int_{q_{2,f}^{cut}}^{q_{1,f}^{cut}} \frac{w + y(q_{2,f}^{high})}{\gamma} f(q_{2,f}^{high}) dq_{2,f}^{high}$$

It is clear that if at least two job openings are drawn above $q_{2,f}^{cut}$, he will get a better offer. The corresponding expected value of high offer is

$$\int_{q_{2,f}^{cut}}^{\overline{q_f}} \frac{w + y(q_{1,f}^{high})}{\gamma} f(q_{2,f}^{high}) dq_{2,f}^{high}$$

Therefore, his cutoff level, $q_{2,f}^{cut}$, also solves to

$$\frac{y(q_{2,f}^{cut})}{\gamma} = V^r(2)$$

where

$$V^r(2) = \begin{cases} \left(C_1^n \left\{ F(q_{1,f}^{cut}) - F(q_{2,f}^{cut}) \right\} F(q_{2,f}^{cut})^{n-1} \right) * \int_{q_{2,f}^{cut}}^{q_{1,f}^{cut}} \frac{w + y(q_{2,f}^{high})}{\gamma} f(q_{2,f}^{high}) dq_{2,f}^{high} \\ + \left(\sum_{t=2}^n C_t^n \left(1 - F(q_{2,f}^{cut}) \right)^t F(q_{2,f}^{cut})^{n-t} \right) * \int_{q_{2,f}^{cut}}^{\overline{q_f}} \frac{w + y(q_{1,f}^{high})}{\gamma} f(q_{2,f}^{high}) dq_{2,f}^{high} \\ \text{when } 0 \leq q_{2,f}^{cut} < q_{1,f}^{cut} \\ \\ \left(\sum_{t=2}^n C_t^n \left(1 - F(q_{2,f}^{cut}) \right)^t F(q_{2,f}^{cut})^{n-t} \right) * \int_{q_{2,f}^{cut}}^{\overline{q_f}} \frac{w + y(q_{2,f}^{high})}{\gamma} f(q_{2,f}^{high}) dq_{2,f}^{high} \\ \text{when } q_{1,f}^{cut} \leq q_{2,f}^{cut} < \overline{q_f} \end{cases}$$

The proposition follows.

Proposition 3 *The probability of successful matching with a better position than $q_{2,f}^{cut}$, $\text{prob}^2(q_{2,f}^{cut})$, decreases in $q_{2,f}^{cut}$ and there exists a unique cutoff level, $q_{2,f}^{cut}$, which guarantees that the 2nd ranked candidate accepts an offer, $q_{2,f}$, if $q_{2,f} \geq q_{2,f}^{cut}$. Otherwise, he waits for a better offer in the future.*

Proof. *Omitted.* ■

3.2.1 The probability of successful matching: $0 \leq q_{k,f} < \text{Min}[q_{j,f}^{\text{cut}}]$

I, below, generalize the probability function⁴ that the k th ranked candidate ($3 \leq k \leq n$)⁵ can get a better offer, $q_{k,f}^{\text{high}} > q_{k,f}^{\text{cut}}$. I, first, consider the case that a favorite offer, $q_{k,f}^{\text{cut}}$, is less than $\text{Min}[q_{j,f}^{\text{cut}}]$, where $\text{Min}[q_{j,f}^{\text{cut}}]$ is the minimum cutoff level of higher ranked candidates than k th ranked candidate, $j = 1, 2, \dots, k-1$. If all candidates with higher ranking than k have an unique cutoff level, then the probability that a k th ranked candidate can get a better offer, $q_{k,f}^{\text{high}} > q_{k,f}^{\text{cut}}$ is defined by

$$\text{prob}^k(q_{k,f}^{\text{cut}}) = G_A^k(q_{k,f}^{\text{cut}}) + G_C^k(q_{k,f}^{\text{cut}}) + G_D^k(q_{k,f}^{\text{cut}}) \text{ if } 0 \leq q_{k,f}^{\text{cut}} < \text{Min}[q_{j,f}^{\text{cut}}] \quad (5)$$

where

$$\begin{aligned} G_A^k(q_{k,f}^{\text{cut}}) &= \sum_{i=z}^{k-1} \sum_{t=z-1}^{k-i} C_{t,i-1}^n \left(F\left((z-1)^{\text{th}} \text{Min}[q_{j,f}^{\text{cut}}]\right) - F\left((z-2)^{\text{th}} \text{Min}[q_{j,f}^{\text{cut}}]\right) \right)^{i-1} \\ &\quad * \left(1 - F\left((z-1)^{\text{th}} \text{Min}[q_{j,f}^{\text{cut}}]\right) \right)^t F(q_{k,f}^{\text{cut}})^{n-t-i+1} \\ &\quad + \sum_{t=z}^{k-1} C_t^n \left(F\left((z-1)^{\text{th}} \text{Min}[q_{j,f}^{\text{cut}}]\right) - F\left((z-2)^{\text{th}} \text{Min}[q_{j,f}^{\text{cut}}]\right) \right)^t F(q_{k,f}^{\text{cut}})^{n-t} \end{aligned}$$

where $z = 2, \dots, k-1$

$$G_C^k(q_{k,f}) = C_{k-1}^n \left(F\left((k-1)^{\text{th}} \text{Min}[q_{j,f}^{\text{cut}}]\right) - F\left((k-2)^{\text{th}} \text{Min}[q_{j,f}^{\text{cut}}]\right) \right)^{k-1} F(q_{k,f}^{\text{cut}})^{n-k+1}$$

and

$$G_D^k(q_{k,f}^{\text{cut}}) = \sum_{t=k}^n C_t^n \left(1 - F(q_{k,f}^{\text{cut}}) \right)^t F(q_{k,f}^{\text{cut}})^{n-t}$$

$\text{Min}[q_{j,f}^{\text{cut}}]$ ($= 1^{\text{st}} \text{Min}[q_{j,f}^{\text{cut}}]$) is the minimum cutoff level, $j = 1, 2, \dots, k-1$ and $z^{\text{th}} \text{Min}[q_{j,f}^{\text{cut}}]$ is the z^{th} minimum cutoff level among $q_{j,f}^{\text{cut}}$, $j = 1, 2, \dots, k-1$. For the simplicity, let $0^{\text{th}} \text{Min}[q_{j,f}^{\text{cut}}] = q_{k,f}^{\text{cut}}$. Even though less than k job openings are drawn from the space above $q_{k,f}^{\text{cut}}$, there is the possibility that he will get a better offer in the future. $G_A^k(q_{k,f}^{\text{cut}})$ and $G_C^k(q_{k,f}^{\text{cut}})$ show the probability that these cases would occur. For instance, if only one job is drawn from the space between $q_{k,f}^{\text{cut}}$ and $\text{Min}[q_{j,f}^{\text{cut}}]$ and other $n-1$ job are drawn from the space below $q_{k,f}^{\text{cut}}$, the k^{th} ranked candidate will get a better offer in the future because all higher ranked candidates than the k^{th} ranked one will reject this offer. Suppose only two jobs are drawn from the space between $\text{Min}[q_{j,f}^{\text{cut}}]$ and $2^{\text{nd}} \text{Min}[q_{j,f}^{\text{cut}}]$ and other $n-2$ job are drawn from the space below $q_{k,f}^{\text{cut}}$. Then, he can also get a better offer because all higher ranked candidates except the candidate who has $\text{Min}[q_{j,f}^{\text{cut}}]$ quality level will reject these offers. There are many different cases which guarantee that the k^{th} ranked candidate will get a better offer in the future even though less than k job openings

⁴You can see more details in Appendix.

⁵As I mentioned before, I will focus on the cutoff level of the first n number of candidates in the pool of potential candidates because we can generally guarantee that the first n number of candidates will surely have received an offer in this model.

are drawn from the space above $q_{k,f}^{cut}$. More explanations are given in the appendix. It is straightforward that if at least k jobs are drawn from the space above $q_{k,f}^{cut}$, the k^{th} ranked candidate gets a better offer. $G_D^k(q_{k,f}^{cut})$ shows the probability that this case would occur. Now, I will show more explicit form of the expected flow value of getting a better offer,

$$prob^k(q_{k,f}^{cut})EV^a(q_{k,f}^{high}), \text{ if } 0 \leq q_{k,f}^{cut} < Min[q_{j,f}^{cut}]$$

in Equation (2) by

$$\begin{aligned} prob^k(q_{k,f}^{cut})EV^a(q_{k,f}^{high}) &= G_A^k(q_{k,f}^{cut}) * E_A V^a(q_{k,f}^{high}) + G_C^k(q_{k,f}^{cut}) * E_C V^a(q_{k,f}^{high}) \\ &\quad + G_D^k(q_{k,f}^{cut}) * E_D V^a(q_{k,f}^{high}) \text{ if } 0 \leq q_{k,f}^{cut} < Min[q_{j,f}^{cut}] \end{aligned}$$

where

$$E_A V^a(q_{k,f}^{high}) = \int_{(z-2)^{th} Min[q_{j,f}^{cut}]}^{(z-1)^{th} Min[q_{j,f}^{cut}]} \frac{w + y(q_{k,f}^{high})}{\gamma} f(q_{k,f}^{high}) dq_{k,f}^{high}, \quad z = 2, \dots, k-1$$

$$E_C V^a(q_{k,f}^{high}) = \int_{(k-2)^{th} Min[q_{j,f}^{cut}]}^{(k-1)^{th} Min[q_{j,f}^{cut}]} \frac{w + y(q_{k,f}^{high})}{\gamma} f(q_{k,f}^{high}) dq_{k,f}^{high}$$

and

$$E_D V^a(q_{k,f}^{high}) = \int_{q_{k,f}}^{\bar{q}} \frac{w + y(q_{k,f}^{high})}{\gamma} f(q_{k,f}^{high}) dq_{k,f}^{high}$$

$E_A V^a(q_{k,f}^{high})$, $E_C V^a(q_{k,f}^{high})$ and $E_D V^a(q_{k,f}^{high})$ represent the expected flow value of successful matching with a higher quality position than $q_{k,f}^{cut}$, which are mapped with the expected range of $q_{k,f}^{high}$, respectively.

3.2.2 The probability of successful matching: $Min[q_{j,f}^{cut}] \leq q_{k,f}^{cut} < \bar{q}_f$

Second, I focus on the case that a favorite offer, $q_{k,f}^{cut}$, is greater than $Min[q_{j,f}^{cut}]$, where $Min[q_{j,f}^{cut}]$ is the minimum cutoff level, $j = 1, 2, \dots, k-1$. Notice that

$$Min[q_{j,f}^{cut}] = 1^{st} Min[q_{j,f}^{cut}]$$

and

$$(k-1)^{th} Min[q_{j,f}^{cut}] = Max[q_{j,f}^{cut}]$$

By similar methods, the probability ⁶that a k th ranked candidate ($k \geq 3$) can get a better offer, $q_{k,f}^{high} > q_{k,f}^{cut}$, is defined by

$$prob^k(q_{k,f}^{cut}) = \left\{ \begin{array}{l} G_F^k(q_{k,f}^{cut}) + G_H^k(q_{k,f}^{cut}) \\ \text{when } (z-2)^{th} Min[q_{j,f}^{cut}] \leq q_{k,f}^{cut} < (z-1)^{th} Min[q_{j,f}^{cut}], \quad z = 3, \dots, k-1 \\ \\ G_G^k(q_{k,f}) + G_H^k(q_{k,f}) \\ \text{when } (k-2)^{th} Min[q_{j,f}^{cut}] \leq q_{k,f}^{cut} < (k-1)^{th} Min[q_{j,f}^{cut}] = Max[q_{j,f}^{cut}] \\ \\ G_H^k(q_{k,f}), \text{ when } q_{k,f}^{cut} \geq (k-1)^{th} Min[q_{j,f}^{cut}] = Max[q_{j,f}^{cut}] \end{array} \right\}$$

where

$$G_F^k(q_{k,f}^{cut}) = \left\{ \begin{array}{l} \sum_{i=z}^{k-1} \sum_{t=1}^{k-i} C_{t,i-1}^n \left(F((z-1)^{th} Min[q_{j,f}^{cut}]) - F(q_{k,f}^{cut}) \right)^{i-1} \\ * \left(1 - F((z-1)^{th} Min[q_{j,f}^{cut}]) \right)^t F(q_{k,f}^{cut})^{n-t-i+1} \\ + \sum_{t=z-1}^{k-1} C_t^n \left(F((z)^{th} Min[q_{j,f}^{cut}]) - F(q_{k,f}^{cut}) \right)^t F(q_{k,f}^{cut})^{n-t} \end{array} \right\}$$

$$G_G^k(q_{k,f}^{cut}) = C_{k-1}^n \left(F((k-1)^{th} Min[q_{j,f}^{cut}]) - F(q_{k,f}) \right)^{k-1} F(q_{k,f})^{n-k+1}$$

and

$$G_H^k(q_{k,f}^{cut}) = \sum_{t=k}^n C_t^n (1 - F(q_{k,f}^{cut}))^t F(q_{k,f}^{cut})^{n-t}$$

Notice that when $k = 3$, we do not have to take into account the first range ($(z-2)^{th} Min[q_{j,f}^{cut}] \leq q_{k,f}^{cut} < (z-1)^{th} Min[q_{j,f}^{cut}]$). Suppose his cutoff level is on $Min[q_{j,f}^{cut}] \leq q_{k,f}^{cut} < 2^{nd} Min[q_{j,f}^{cut}]$. Only two job openings are drawn from the space between $q_{k,f}^{cut}$ and $2^{nd} Min[q_{j,f}^{cut}]$ and other $n-2$ jobs are drawn from space below $q_{k,f}^{cut}$. Then, he will get a better offer because all higher ranked candidates except the candidate who has $Min[q_{j,f}^{cut}]$ will reject these offers. We can generalize these cases in the following manner. Even though less than k job openings are drawn from the space above $q_{k,f}^{cut}$ the k th ranked candidate ($k \geq 3$) can get a better offer when at least two job are drawn from the space between $q_{k,f}^{cut}$ and $2^{nd} Min[q_{j,f}^{cut}]$. $G_F^k(q_{k,f}^{cut})$ and $G_G^k(q_{k,f}^{cut})$ show the probability of these cases. It is clear that at least k number of job opening are drawn from the space above $q_{k,f}^{cut}$ then he will get a better offer. $G_H^k(q_{k,f}^{cut})$ provides the probability function of this case. Similarly, we can find more explicit form of the expected flow value of getting a better offer, $prob^k(q_{k,f}^{cut}) EV^a(q_{k,f}^{high})$, ($k \geq 3$) in Equation (2) by

⁶See Appendix for more details.

$$\begin{aligned}
\text{prob}^k(q_{k,f}^{cut})EV^a(q_{k,f}^{high}) = & \left\{ \begin{aligned} & G_F^k(q_{k,f}^{cut}) * \left\{ \int_{q_{k,f}^{cut}}^{(z-2)^{th} Min[q_{j,f}^{cut}]} \frac{w+\eta q_{k,f}^{high}}{\gamma} f(q_{k,f}^{high}) dq_{k,f}^{high} \right\} \\ & + G_H^k(q_{k,f}^{cut}) * \left\{ \int_{q_{k,f}^{cut}}^{\bar{q}} \frac{w+\eta q_{k,f}^{high}}{\gamma} f(q_{k,f}^{high}) dq_{k,f}^{high} \right\} \\ & \text{when } (z-1)^{th} Min[q_{j,f}^{cut}] \leq q_{k,f}^{cut} < (z-2)^{th} Min[q_{j,f}^{cut}], \quad z = 3, \dots, k-1 \\ & G_G^k(q_{k,f}^{cut}) * \left\{ \int_{q_{k,f}^{cut}}^{(k-1)^{th} Min[q_{j,f}^{cut}]} \frac{w+\eta q_{k,f}^{high}}{\gamma} f(q_{k,f}^{high}) dq_{k,f}^{high} \right\} \\ & + G_H^k(q_{k,f}^{cut}) * \left\{ \int_{q_{k,f}^{cut}}^{\bar{q}} \frac{w+\eta q_{k,f}^{high}}{\gamma} f(q_{k,f}^{high}) dq_{k,f}^{high} \right\} \\ & \text{when } (k-2)^{th} Min[q_{j,f}^{cut}] \leq q_{k,f}^{cut} < (k-1)^{th} Min[q_{j,f}^{cut}] = Max[q_{j,f}^{cut}] \\ & G_H^k(q_{k,f}^{cut}) * \left\{ \int_{q_{k,f}^{cut}}^{\bar{q}} \frac{w+\eta q_{k,f}^{high}}{\gamma} f(q_{k,f}^{high}) dq_{k,f}^{high} \right\} \\ & \text{when } q_{k,f}^{cut} \geq (k-1)^{th} Min[q_{j,f}^{cut}] = Max[q_{j,f}^{cut}] \end{aligned} \right\}
\end{aligned}$$

4 Numerical Analysis

4.1 The quality of firm: RANKING

I propose a numerical analysis and calibration of the model. *Table 1* shows the baseline parameter and functional form.

Table 1

I assume that the reputational value function, $y(q_{k,f})$, is linear in the quality of firm and the distribution of firms' quality, $F(q_{k,f})$, follows a uniform distribution on $[0, 1]$. Here, we can interpret the quality of firm as the ranking of firm or quantile of ranking in the economy. The results of the simulations are shown in *Table 2 and Figure 1*.

Table 2/Figure 1

Table 2 shows the cutoff firm quality level of candidates for outside directors by the ranking based on the baseline parameter and functional forms. The cutoff value of the first ranked candidate is 0.4. We can interpret this value as the quantile of firm ranking in the economy. If we assume that there are 250 firms in the economy, this value implies the 150th ranked firm. The notable feature is that the relationship between the ranking of candidates and the ranking of firms is positive and concave. It means that higher ranked (better talented) candidates have the higher cutoff ranking of firms, but the marginal

increase in the ranking of firm is diminishing. For instance, the difference between the cutoff level of 1st ranked candidate and 20th ranked one is 10 in terms of firm ranking, but the difference between 20th ranked candidate and 30 ranked one is 64 in terms of firm ranking. Given the number of job opening, n , is large enough, the reason is that the gap in the probability of successful matching with the better offer in the future among high ranked candidates are arbitrary small in a low range of $q_{k,f}$, which makes a small difference in the cutoff level, but this gap gets bigger when the ranking becomes lower. From the viewpoints of high ranked candidates, if the quality of current offer is relatively small, they expect that they would get a better offer in the future with the probability which is almost close to 1. *Figure 2* shows the probability of successful matching in the future by the rank of candidates.

Figure 2

Table 3-A shows the sensitivity of the cutoff level to the change in the discounted factor, γ . In a case of the first ranked candidate, the cutoff firm ranking level decrease in the discounted factor, but the marginal effect of the discounted factor is diminished. For instance, the increase from $\gamma = 0.05$ to $\gamma = 0.1$ decreases the firm ranking by 3.25, but the increase from $\gamma = 0.25$ to $\gamma = 0.3$ decreases the firm ranking by 2.23. The interpretation is clear. As the candidate become less patient, his cutoff level decreases. *Table 3-B* provides the effect of change in the functional form of reputation value generated by outside directorships on the cutoff level of the 1st ranked candidate. When $y(q_f) = \eta q_f^2$, the cutoff level is the 120 ranked firm over 250 firms in the economy, which is compared to the 150 ranked firm in the linear form, $y(q_f) = \eta q_f$ case. Since the marginal reputation value increases in q_f under the functional form, $y(q_f) = \eta q_f^2$, the highly ranked candidates tend to wait for a better offer (better quality) in the future to enjoy it.

Table 3

4.2 The quality of firm: the quality of CEO

Here, I assume that the quality of firm, $F(q_f)$, follows the distribution of CEO talent inferred by Gabaix and Landier (2006). Their calibration shows that there is an upper bound $T_{\max}$, in the distribution of talent and in the upper tail, talent density is

$$P(T > t) = B'(T_{\max} - t)^{\frac{3}{2}} \text{ for } t \text{ close to } T_{\max}$$

It follows that the density, left of the upper bound $T_{\max}$, is

$$f(t) = \frac{3B'}{2}(T_{\max} - T)^{\frac{1}{2}}, \text{ for } t \text{ close to } T_{\max}^7$$

⁷See Gabaix and Landier (2006) (page 24).

I approximate the upper tail distribution of talent $F(q_f)$ by

$$F(q_f) = -B'(\overline{q_f} - q_f)^{\frac{1}{2}} + 1 \text{ on } [0, \overline{q_f}], \text{ where } F(0) = 0 \text{ and } F(\overline{q_f}) = 1$$

Figure 3/Table 4

Figure 3 shows the shape of the distribution, $f(q_f)$ and *Table 4* provides the cutoff firm quality level of candidates for outside directors by the ranking under the assumption of $F(q_f)$. In this case, I also use the baseline parameter and linear form of reputation function. If we assume that there are 250 firms in the economy, the cutoff level of the first ranked candidate is the 140th ranked firm. The relationship between the ranking of candidates and the ranking of firms is still positive and concave. For instance, the difference between the cutoff level of 1st ranked candidate and 20th ranked one is 10 in terms of firm ranking, but the difference between 20th ranked candidate and 30 ranked one is 68.

5 Conclusion

I construct a matching model ("take it or leave it offer") in a frictional directorship market to explain the (observed) unbalanced match between the quality of outside directors on boards and the firms. My calibration shows that if the firm quality follows an uniform distribution, the best candidate would be likely to go to the 150th ranked firm over 250 firms. Also, the best candidate would be likely to be matched with the 140th ranked firms under Gabaix and Landier (2006)'s extreme value distribution.

References

- Gabaix, Xavier and Augustin Landier. 2006. "Why Has CEO Pay Increased So Much?" *Journal of Political Economy*, *Forthcomming*.
- Gale, David and L. S. Shapley. 1962. "College Admissions and the Stability of Marriage." *The American Mathematical Monthly* 69:9–15.
- McCall, John. 1970. "Economics of Information and Job Search." *Quarterly Journal of Economics* 84(1):113–126.

6 Appendix

Proof. of the Proposition 1: By assumption, $V^a(k, 0) = 0$ and $V^r(k, 0) > 0$. Also, $V^a(k, \bar{q}_f) > 0$ and $V^r(k, \bar{q}_f) = 0$. Since $\frac{\partial V^a(k, q_{k,f})}{\partial q_{k,f}} > 0$ and $\frac{\partial V^r(k, q_{k,f})}{\partial q_{k,f}} \leq 0$ (by assumption), there exists a unique cutoff level. ■

Proof. of the Proposition 2: (1) $prob^1(q_{1,f}) = \sum_{t=1}^n C_t (1 - F(q_{1,f}))^t F(q_{1,f})^{n-t} = 1 - F(q_{1,f})^n$. So, $\frac{\partial prob^1(q_{1,f})}{\partial q_{1,f}} = \frac{\partial (1 - F(q_{1,f})^n)}{\partial q_{1,f}} \leq 0$. (2) $EV^a(1, 0, q_{k,f}^{high}) = \int_0^{\bar{q}_f} \frac{y(q_{1,f}^{high})}{\gamma} f(q_{1,f}^{high}) dq_{1,f}^{high} > 0$ and

$$\begin{aligned} \frac{\partial V^r(1, q_{k,f})}{\partial q_{1,f}} &= \frac{\partial \left\{ \frac{prob^1(q_{1,f}) \int_{q_{1,f}}^{\bar{q}_f} \frac{y(q_{1,f}^{high})}{\gamma} f(q_{1,f}^{high}) dq_{1,f}^{high}}{r + (prob^1(q_{1,f}))} \right\}}{\partial q_{1,f}} \\ &= \frac{\left\{ \frac{\partial prob^1(q_{1,f})}{\partial q_{1,f}} \left(\int_{q_{1,f}}^{\bar{q}_f} \frac{y(q_{1,f}^{high})}{\gamma} f(q_{1,f}^{high}) dq_{1,f}^{high} \right) + prob^1(q_{1,f}) \left(-\frac{y(q_{1,f}^{high})}{\gamma} f(q_{1,f}^{high}) \right) \right\} \{r + prob^1(q_{1,f})\}}{\{r + (prob^1(q_{1,f}))\}^2} \\ &\quad - \frac{prob^1(q_{1,f}) \int_{q_{1,f}}^{\bar{q}_f} \frac{y(q_{1,f}^{high})}{\gamma} f(q_{1,f}^{high}) dq_{1,f}^{high} \left(\frac{\partial prob^1(q_{1,f})}{\partial q_{1,f}} \right)}{\{r + (prob^1(q_{1,f}))\}^2} \\ &= \frac{\left\{ \frac{\partial prob^1(q_{1,f})}{\partial q_{1,f}} \left(\int_{q_{1,f}}^{\bar{q}_f} \frac{y(q_{1,f}^{high})}{\gamma} f(q_{1,f}^{high}) dq_{1,f}^{high} \right) + prob^1(q_{1,f}) \left(-\frac{y(q_{1,f}^{high})}{\gamma} f(q_{1,f}^{high}) \right) \right\} r}{\{r + (prob^1(q_{1,f}))\}^2} \\ &\quad + \frac{\left\{ prob^1(q_{1,f}) \left(-\frac{y(q_{1,f}^{high})}{\gamma} f(q_{1,f}^{high}) \right) \right\} prob^1(q_{1,f})}{\{r + (prob^1(q_{1,f}))\}^2} \leq 0 \end{aligned}$$

Then, the proposition 1 is satisfied, which implies that there exist a unique cutoff level. ■

6.1 The probability that a k th ranked candidate can get a better offer,

$$q_{k,f}^{high} \geq q_{k,f}$$

Here, I show how to derive the probability that a k th ranked candidate can get a better offer, $q_{k,f}^{high} \geq q_{k,f}$. Let me explain the outline by showing the case of the 5th ranked candidate.

Case 1: the 5th ranked candidate: $q_{5,f} < \text{Min}[q_{1,f}^{cut}, q_{2,f}^{cut}, q_{3,f}^{cut}, q_{4,f}^{cut}]$

1. Suppose his current offer, $q_{5,f}$, is less than the minimum cutoff level of higher ranked candidates, $q_{5,f} < \text{Min}[q_{1,f}^{cut}, q_{2,f}^{cut}, q_{3,f}^{cut}, q_{4,f}^{cut}]$. Then, if at least five (k) job

openings out of n number job openings are drawn from the space above $q_{5,f}$, he will get a better offer in the future. The probability in this case is that

$$prob_a^5(q_{5,f}) = \sum_{t=5}^n C_t^n (1 - F(q_{5,f}))^t F(q_{5,f})^{n-t}$$

$$\text{where } C_t^n = \binom{n}{t}$$

C_t^n represents the distinguishible permutations of n objects, t of one type and $n - t$ of another type (binomial coefficient).

2. From here, suppose that less than 5 (k) jobs are drawn from the space above $q_{5,f}$. If only four job openings out of n number job openings are drawn from the space between $3^{rd}Min[q_{1,f}^{cut}, q_{2,f}^{cut}, q_{3,f}^{cut}, q_{4,f}^{cut}]$ and $4^{th}Min[q_{1,f}^{cut}, q_{2,f}^{cut}, q_{3,f}^{cut}, q_{4,f}^{cut}] = Max[q_{1,f}^{cut}, q_{2,f}^{cut}, q_{3,f}^{cut}, q_{4,f}^{cut}]$, he will get a better offer because the candidate who has the maximum cutoff level will reject all offers. The probability is

$$prob_b^5(q_{5,f}) = C_4^n \left(F(Max) - F(3^{rd}Min) \right)^4 F(q_{5,f})^{n-4}$$

3. If three or four job opening are drawn from the space between $2^{nd}Min[q_{1,f}^{cut}, q_{2,f}^{cut}, q_{3,f}^{cut}, q_{4,f}^{cut}]$ and $3^{rd}Min[q_{1,f}^{cut}, q_{2,f}^{cut}, q_{3,f}^{cut}, q_{4,f}^{cut}]$, then he also can get a better offer with the probability

$$prob_c^5(q_{5,f}) = C_3^n \left(F(3^{rd}Min) - F(2^{nd}Min) \right)^3 F(q_{5,f})^{n-3}$$

$$+ C_4^n \left(F(3^{rd}Min) - F(2^{nd}Min) \right)^4 F(q_{5,f})^{n-4}$$

$$+ C_{3,1}^n \left(F(3^{rd}Min) - F(2^{nd}Min) \right)^3 * \left(F(\overline{q_f}) - F(3^{rd}Min) \right)^1 F(q_{5,f})^{n-4}$$

$$\text{where } C_{a,b}^n = \binom{n}{a, b, n-a-b}$$

because both candidates who have the maximum cutoff level and $3^{rd}Min[q_{1,f}^{cut}, q_{2,f}^{cut}, q_{3,f}^{cut}, q_{4,f}^{cut}]$ will reject all offers in the case of first two terms in the probability function and one of them will reject all offers in the final case. $C_{a,b}^n$ represents the distinguishible permutations of n objects, a of one type, b of second type and $n - a - b$ of third type (multinomial coefficient).

4. If two, three or four job opening are drawn from the space between $Min[q_{1,f}^{cut}, q_{2,f}^{cut}, q_{3,f}^{cut}, q_{4,f}^{cut}] = 1^{st}Min[q_{1,f}^{cut}, q_{2,f}^{cut}, q_{3,f}^{cut}, q_{4,f}^{cut}]$ and $2^{nd}Min[q_{1,f}^{cut}, q_{2,f}^{cut}, q_{3,f}^{cut}, q_{4,f}^{cut}]$, then he also can get

a better offer with the probability

$$\begin{aligned}
prob_d^5(q_{5,f}) &= C_2^n \left(F(2^{nd}Min) - F(Min) \right)^2 F(q_{5,f})^{n-2} \\
&+ C_3^n \left(F(2^{nd}Min) - F(Min) \right)^3 F(q_{5,f})^{n-3} \\
&+ C_4^n \left(F(2^{nd}Min) - F(Min) \right)^4 F(q_{5,f})^{n-4} \\
&+ C_{2,1}^n \left(F(2^{nd}Min) - F(Min) \right)^2 * \left(F(\overline{q_f}) - F(2^{nd}Min) \right)^1 F(q_{5,f})^{n-3} \\
&+ C_{2,2}^n \left(F(2^{nd}Min) - F(Min) \right)^2 * \left(F(\overline{q_f}) - F(2^{nd}Min) \right)^2 F(q_{5,f})^{n-4}
\end{aligned}$$

5. Finally, if one, two, three or four job opening are drawn from the space between $q_{5,f}$ and $Min[q_{1,f}^{cut}, q_{2,f}^{cut}, q_{3,f}^{cut}, q_{4,f}^{cut}] = 1^{st} Min[q_{1,f}^{cut}, q_{2,f}^{cut}, q_{3,f}^{cut}, q_{4,f}^{cut}]$, then he also can get a better offer with the probability

$$\begin{aligned}
prob_g^5(q_{5,f}) &= C_1^n (F(Min) - F(q_{5,f}))^1 F(q_{5,f})^{n-1} \\
&+ C_2^n (F(Min) - F(q_{5,f}))^2 F(q_{5,f})^{n-2} \\
&+ C_3^n (F(Min) - F(q_{5,f}))^3 F(q_{5,f})^{n-3} \\
&+ C_4^n (F(Min) - F(q_{5,f}))^4 F(q_{5,f})^{n-4} \\
&+ C_{1,1}^n (F(Min) - F(q_{5,f}))^1 * (F(\overline{q_f}) - F(Min))^1 F(q_{5,f})^{n-2} \\
&+ C_{1,2}^n (F(Min) - F(q_{5,f}))^1 * \left(F(\overline{q_f}) - F(2^{nd}Min) \right)^2 F(q_{5,f})^{n-3} \\
&+ C_{1,3}^n (F(Min) - F(q_{5,f}))^1 * (F(\overline{q_f}) - F(Min))^3 F(q_{5,f})^{n-4}
\end{aligned}$$

Case 2: the 5th ranked candidate: $Min[q_{1,f}^{cut}, q_{2,f}^{cut}, q_{3,f}^{cut}, q_{4,f}^{cut}] \leq q_{5,f} < 2^{nd} Min[q_{1,f}^{cut}, q_{2,f}^{cut}, q_{3,f}^{cut}, q_{4,f}^{cut}]$

1. Suppose his current offer, $q_{5,f}$, is $Min[q_{1,f}^{cut}, q_{2,f}^{cut}, q_{3,f}^{cut}, q_{4,f}^{cut}] \leq q_{5,f} < 2^{nd} Min[q_{1,f}^{cut}, q_{2,f}^{cut}, q_{3,f}^{cut}, q_{4,f}^{cut}]$ and less than k job openings are drawn from the space above $q_{5,f}$. Then, if two, three or four job openings out of n number job openings are drawn from the space between his current offer and $2^{nd} Min[q_{1,f}^{cut}, q_{2,f}^{cut}, q_{3,f}^{cut}, q_{4,f}^{cut}]$, he will get a better offer.

The probability is that

$$\begin{aligned}
prob_h^5(q_{5,f}) &= C_2^n \left(F(2^{nd} Min) - F(q_{5,f}) \right)^2 F(q_{5,f})^{n-2} \\
&+ C_3^n \left(F(2^{nd} Min) - F(q_{5,f}) \right)^3 F(q_{5,f})^{n-3} \\
&+ C_4^n \left(F(2^{nd} Min) - F(q_{5,f}) \right)^4 F(q_{5,f})^{n-4} \\
&+ C_{2,1}^n \left(F(2^{nd} Min) - F(q_{5,f}) \right)^2 * \left(F(\overline{q_f}) - F(2^{nd} Min) \right)^1 F(q_{5,f})^{n-3} \\
&+ C_{2,2}^n \left(F(2^{nd} Min) - F(q_{5,f}) \right)^2 * \left(F(\overline{q_f}) - F(2^{nd} Min) \right)^2 F(q_{5,f})^{n-4}
\end{aligned}$$

2. Also, if at least five (k) job openings out of n number job openings are drawn from the space above $q_{5,f}$, he will get a better offer in the future. The probability is that

$$prob_i^5(q_{5,f}) = \sum_{t=5}^n C_t^n (1 - F(q_{5,f}))^t F(q_{5,f})^{n-t}$$

Case 3: the 5th ranked candidate: $2^{nd} Min[q_{1,f}^{cut}, q_{2,f}^{cut}, q_{3,f}^{cut}, q_{4,f}^{cut}] \leq q_{5,f} < 3^{rd} Min[q_{1,f}^{cut}, q_{2,f}^{cut}, q_{3,f}^{cut}, q_{4,f}^{cut}]$

1. Also, suppose that less than 5 (k) job openings are drawn from the space above $q_{5,f}$. In this case, if three or four job openings out of n number job openings are drawn from the space between his current offer and $3^{rd} Min[q_{1,f}^{cut}, q_{2,f}^{cut}, q_{3,f}^{cut}, q_{4,f}^{cut}]$ he will get a better offer with probability

$$\begin{aligned}
prob_j^5(q_{5,f}) &= C_3^n \left(F(3^{rd} Min) - F(q_{5,f}) \right)^3 F(q_{5,f})^{n-3} \\
&+ C_4^n \left(F(3^{rd} Min) - F(q_{5,f}) \right)^4 F(q_{5,f})^{n-4} \\
&+ C_{3,1}^n \left(F(3^{rd} Min) - F(q_{5,f}) \right)^3 * \left(F(\overline{q_f}) - F(3^{rd} Min) \right)^1 F(q_{5,f})^{n-4}
\end{aligned}$$

2. Also, if at least five (k) job openings out of n number job openings are drawn from the space above $q_{5,f}$ he will get a better offer in the future. The probability is that

$$prob_i^5(q_{5,f}) = \sum_{t=5}^n C_t^n (1 - F(q_{5,f}))^t F(q_{5,f})^{n-t}$$

Case 4: the 5th ranked candidate: $3^{rd} Min[q_{1,f}^{cut}, q_{2,f}^{cut}, q_{3,f}^{cut}, q_{4,f}^{cut}] \leq q_{5,f} < 4^{th} Min[q_{1,f}^{cut}, q_{2,f}^{cut}, q_{3,f}^{cut}, q_{4,f}^{cut}] = Max[q_{1,f}^{cut}, q_{2,f}^{cut}, q_{3,f}^{cut}, q_{4,f}^{cut}]$

1. Also, suppose that less than k job openings are drawn from the space above $q_{5,f}$. If four job openings out of n number job openings are drawn from the space between his current offer and $Max[q_{1,f}^{cut}, q_{2,f}^{cut}, q_{3,f}^{cut}, q_{4,f}^{cut}]$ he will get a better offer with probability

$$prob_l^5(q_{5,f}) = C_4^n (F(Max) - F(q_{5,f}))^4 F(q_{5,f})^{n-4}$$

2. Also, if at least five (k) job openings out of n number job openings are drawn from the space above $q_{5,f}$, he will get a better offer in the future. The probability is that

$$prob_i^5(q_{5,f}) = \sum_{t=5}^n C_t^n (1 - F(q_{5,f}))^t F(q_{5,f})^{n-t}$$

Case 5: the 5th ranked candidate: $Max[q_{1,f}^{cut}, q_{2,f}^{cut}, q_{3,f}^{cut}, q_{4,f}^{cut}] \leq q_{5,f} < \overline{q_f}$ In this case, he will get a better offer in the future only if at least five (k) job openings out of n number job openings are drawn from the space above $q_{5,f}$. It is given by

$$prob_i^5(q_{5,f}) = \sum_{t=5}^n C_t^n (1 - F(q_{5,f}))^t F(q_{5,f})^{n-t}$$

Overally, the probability that the 5th ranked candidate can get a better offer, $q_{k,f}^{high} \geq q_{k,f}$, in the future is

$$prob^5(q_{5,f}) = \left\{ \begin{array}{l} prob_a^5(q_{3,f}) + prob_b^5(q_{3,f}) + prob_c^5(q_{3,f}) + prob_d^5(q_{3,f}) + prob_g^5(q_{3,f}) \\ \\ when \ q_{5,f} < Min[q_{1,f}^{cut}, q_{2,f}^{cut}, q_{3,f}^{cut}, q_{4,f}^{cut}] \\ \\ prob_h^5(q_{5,f}) + prob_i^5(q_{5,f}) \\ \\ when \ Min[q_{1,f}^{cut}, q_{2,f}^{cut}, q_{3,f}^{cut}, q_{4,f}^{cut}] \leq q_{5,f} < 2^{nd} Min[q_{1,f}^{cut}, q_{2,f}^{cut}, q_{3,f}^{cut}, q_{4,f}^{cut}] \\ \\ prob_j^5(q_{5,f}) + prob_i^5(q_{5,f}) \\ \\ 2^{nd} Min[q_{1,f}^{cut}, q_{2,f}^{cut}, q_{3,f}^{cut}, q_{4,f}^{cut}] \leq q_{5,f} < 3^{rd} Min[q_{1,f}^{cut}, q_{2,f}^{cut}, q_{3,f}^{cut}, q_{4,f}^{cut}] \\ \\ prob_l^5(q_{5,f}) + prob_i^5(q_{5,f}) \\ \\ 3^{rd} Min[q_{1,f}^{cut}, q_{2,f}^{cut}, q_{3,f}^{cut}, q_{4,f}^{cut}] \leq q_{5,f} < 4^{th} Min[q_{1,f}^{cut}, q_{2,f}^{cut}, q_{3,f}^{cut}, q_{4,f}^{cut}] = Max[q_{1,f}^{cut}, q_{2,f}^{cut}, q_{3,f}^{cut}, q_{4,f}^{cut}] \\ \\ prob_i^5(q_{5,f}) \ when \ Max[q_{1,f}^{cut}, q_{2,f}^{cut}, q_{3,f}^{cut}, q_{4,f}^{cut}] \leq q_{5,f} < \overline{q_f} \end{array} \right\}$$

Table 1 the baseline parameter and the functional form

<i>Baseline parameter and Functional form</i>	
<i>Baseline parameter</i>	
η : Sensitivity to the quality of firm	10
γ : discount factor	0.05
n : number of job openings at a given time	30
<i>Functional form</i>	
$y(q_k)$: reputation value generated by outside directorship	$y(q_k) = \eta q_k$
$F(q_k)$: the distribution of firm quality	Uniform distribution on $[0,1]$

Table 2 the cutoff firm quality level of candidates for outside director: baseline parameter case

<i>Ranking of candidates</i>	<i>Cutoff level : firm ranking in percentage</i>	<i>Cutoff level: firm ranking (Given N=250), N: Total number of firm in the economy</i>
<i>1st</i>	<i>Lower 40%</i>	<i>150</i>
<i>2nd</i>	<i>Lower 40%</i>	<i>150</i>
<i>3rd</i>	<i>Lower 40%</i>	<i>150</i>
<i>4th</i>	<i>Lower 40%</i>	<i>150</i>
<i>5th</i>	<i>Lower 40%</i>	<i>150</i>
<i>...</i>		
<i>10th</i>	<i>≈ Lower 40% (39.9988%)</i>	<i>≈ 150(150.003)</i>
<i>11th</i>	<i>≈ Lower 40% (39.9961%)</i>	<i>≈ 150(150.00975)</i>
<i>12th</i>	<i>≈ Lower 40% (39.9885%)</i>	<i>≈ 150(150.02875)</i>
<i>...</i>		
<i>20th</i>	<i>Lower 36.61%</i>	<i>≈ 160</i>
<i>...</i>		
<i>25th</i>	<i>Lower 24.98%</i>	<i>≈ 188</i>
<i>...</i>		
<i>30th</i>	<i>Lower 10.39%</i>	<i>≈ 224</i>

Figure 1 the cutoff firm quality level of candidates for outside director

- C^k denotes the k th ranked candidate's cost function of one more searching
- B^k denotes the k th ranked candidate's expected benefit of one more searching
- The intersections represent the cutoff level of each candidate

Figure 2 the probability of successful matching in the future by the rank of candidates

- Given a current offer, q_f , each graph represents the probability of successful matching with a better offer in the future
- k denotes the ranking of candidates

Table 3-A the sensitivity analysis: the effect of change in the discounted factor on the cutoff level of the 1st ranked candidate for outside director

γ : discount factor	Cutoff level : firm ranking in percentage	Cutoff level: firm ranking (Given N=250)
0.05	$\approx$ Lower 40%	$\approx$ 150
0.1	$\approx$ Lower 38.7%	$\approx$ 153 (153.25)
0.15	$\approx$ Lower 37.4%	$\approx$ 157 (156.5)
0.2	$\approx$ Lower 36.2%	$\approx$ 160 (159.5)
0.25	$\approx$ Lower 35.1%	$\approx$ 162 (162.25)
0.3	$\approx$ Lower 34.01%	$\approx$ 165 (164.98)

Table 3-B the sensitivity analysis: the effect of change in the function form $y(q_k)$ on the cutoff level of the 1st ranked candidate for outside director

$y(q_k)$: reputation value	Cutoff level : firm ranking in percentage	Cutoff level: firm ranking (Given N=250)
ηq_k^2	Lower 52.2%	119.5

Figure 3 the distribution of firms' quality, $f(q_f)$

Table 4 the cutoff level of candidate for outside director: the extreme value distribution

- $F(q_k)$ follows the distribution of CEO talent inferred by Gabaix, Xavier and

Augustin Landier (2006): $F(q_k) = -B'(\overline{q_f} - q_f)^{\frac{1}{2}} + 1$, on $[0, \overline{q_f}]$

Ranking of candidates	Cutoff level : talent level of CEO in percentage	Cutoff level: firm ranking (Given $N=250$), N : Total number of firm in the economy
1st	$\approx$ Lower 44% (43.58%)(0.63)	≈ 140
2 nd	$\approx$ Lower 44% (43.58%)	≈ 140
3 rd	$\approx$ Lower 44% (43.58%)	≈ 140
4th	$\approx$ Lower 44% (43.58%)	≈ 140
5th	$\approx$ Lower 44% (43.58%)	≈ 140
...		
10th	$\approx$ Lower 44% (43.58%)	≈ 140
...		
20th	$\approx$ Lower 40% (40.03%)	≈ 150
...		
25th	$\approx$ Lower 30% (30.41%)	≈ 175
...		
30th	$\approx$ Lower 13% (13.12%)	≈ 218